Recent observations have indicated that in addition to the classical “inverted-V” type electron acceleration, auroral electrons often have a field-aligned distribution that is broad in energy and sometimes shows time dispersion indicating acceleration at various altitudes up the field line. Such acceleration is not consistent with a purely electrostatic potential drop, and suggests a wave heating of auroral electrons. Alfvén waves have been observed on auroral field lines carrying sufficient Poynting flux to provide energy for such acceleration. Calculations based on the linear kinetic theory of Alfvén waves indicate that Landau damping of these waves can efficiently convert this Poynting flux into field-aligned acceleration of electrons. At high altitudes along auroral field lines that map into the plasma sheet boundary layer, the plasma gradients are relatively weak and the local kinetic theory can describe this wave-particle interaction. At lower altitudes, the gradient in the Alfvén speed becomes significant, and a non-local description must be used. A non-local theory based on a simplified model of the ionospheric Alfvén resonator is presented. For a given field-aligned current, the efficiency of the wave-particle interaction increases with the ratio of the thermal velocity of the electrons to the Alfvén speed at high altitudes. These calculations indicate that wave acceleration of electrons should occur at and above the altitude where the quasi-static potential drops form.

Introduction
The auroral zone is one of the most intriguing regions in the Earth’s magnetosphere. A variety of plasma physics processes occur on auroral field lines, from large-scale MHD phenomena to the microphysics of plasma instabilities, solitary waves, and radio emissions. It is well known that the aurora occurs on magnetic field lines along which field-aligned currents (FAC’s) flow (e.g., Birkeland, 1908; Iijima and Potemra, 1976). It has long been believed that field-aligned potential drops associated with the FAC’s cause the field-aligned acceleration of electrons related to the formation of the aurora (e.g., Winckler et al., 1958; Evans, 1968, 1974; Mozer and Fahleson, 1970; Frank and Ackerson, 1971; Block, 1972; Gurnett and Frank, 1973; Mozer et al., 1977, 1980). Theories to explain the formation of parallel electric fields and auroral acceleration have fallen into two categories: kinetic models that follow the adiabatic motions of auroral electron populations and fluid models that describe the time development of the parallel electric fields. However, most of the kinetic theories for the parallel electric field are quasi-static in nature. While such models show that parallel electric fields can self-consistently exist, they do not describe the time-dependent evolution of the parallel electric field. Fluid models can describe this evolution, but do not directly model the particle acceleration that produces the aurora. Clearly, a synthesis of the two approaches is necessary for a complete understanding of auroral acceleration.

The main features of the classic “inverted-V” particle acceleration can be well described by a quasi-static potential drop; however, some aspects of auroral acceleration cannot be understood in the context of these quasi-steady models. In particular, a number of rocket and satellite observations have identified nearly field-aligned electron beams precipitating into the auroral zone (Johnstone and Winningham, 1982; Arnoldy et al., 1985; Robinson et al., 1989;
McFadden et al., 1986, 1987, 1990, 1998; Clemmons et al., 1994; Lynch et al., 1994, 1999; Knudsen et al., 1998; Chaston et al., 1999, 2000). Upgoing and counterstreaming electrons have also been observed (Sharp et al., 1980; Klumpar and Heikkila, 1982; Burch et al., 1983; Block and Fälthammar, 1990; Boehm et al., 1995), especially in the downward auroral current region (Marklund et al., 1994, 1995, 1997, 2001; Carlson et al., 1998; Ergun et al., 1998). Features in the non-field-aligned part of the distribution function, such as electron conics (Menietti and Burch, 1985; Temerin and Cravens, 1990; André and Eliasson, 1992; Burch, 1995; Eliasson et al., 1996) are also evidence that quasi-static potential drops do not explain all facets of auroral electron acceleration. Thus, it appears that auroral electron acceleration operates in at least two different modes: acceleration in a quasi-static potential drop and time-dependent acceleration associated with wave activity.

Both steady-state and time-dependent auroral acceleration require a source of energy. The Poynting flux associated with the auroral current system transfers energy from the outer magnetosphere (i.e., the magnetopause and/or the plasma sheet) to the auroral acceleration region (e.g., Mozer et al., 1980). Correlations between electric and magnetic field measurements on low-altitude satellites and rockets have indicated that the Poynting flux is mostly downward, although upward Poynting fluxes are sometimes observed (Sugiura et al., 1982; Sugiura, 1984; Primdahl et al., 1987). Measurements of the quasi-static Poynting flux have indicated that fluxes of 1-10 mW/m² are common (Kelley et al., 1991; Gary et al., 1994, 1995; Thayer et al., 1995; Kletzing et al., 1996) and that fluxes of over 40 mW/m² are sometimes observed (Gary et al., 1994). Small Poynting fluxes (< 1 mW/m²) associated with kinetic Alfvén waves have been observed on Freja (Volwerk et al., 1996). Recent observations from Polar (Wygant et al., 2000; Keiling et al., 2000) have shown that the Poynting flux associated with Alfvénic fluctuations at the plasma sheet boundary layer at 4-6 RE have Poynting fluxes of 1-2 mW/m², which map to values of up to 100 mW/m² at ionospheric altitudes. Moreover, these Alfvénic fluctuations also occur on small perpendicular scales, and are associated with field-aligned electron acceleration (Wygant et al., 2002). Observations of large parallel electric fields from FAST also indicate that such fields are found in regions of strong Alfvénic activity (Ergun et al., 2001). Although energy input to the aurora from the kinetic energy of inflowing particles cannot be ignored, it is clear that the electromagnetic energy input is a significant energy source for auroral acceleration, especially during the dynamic processes that first establish the potential drop. The electromagnetic energy input from the outer magnetosphere has recently been emphasized by Haerendel (2000) and Song and Lysak (2001).

Models of the quasi-static process are most often based on the adiabatic motion of charged particles in the magnetic mirror geometry, such as the Knight (1973) relation, which describes the response of the electrons in such a geometry to the presence of a parallel potential drop. Such a model must be combined with a model of quasi-neutrality (e.g., Chiu and Schulz, 1978; Stern, 1981) or a full solution of the Poisson equation (Chiu and Cornwall, 1980) in order to establish a self-consistent model of the potential drop. Such models have had a resurgence recently in light of results from the FAST satellite (Temerin and Carlson, 1998; Rönnmark, 1999; Schriffer, 1999; Jasperse et al., 2000). A detailed model of this sort has been presented by Ergun et al. (2000), who used 8 populations of ionospheric and magnetospheric particles in a Vlasov model to construct a self-consistent potential profile. This model shows that one or two narrow-scale transition layers exist with large, localized parallel electric fields.

On a more localized scale, the development of parallel potential drops has been studied by the use of BGK solutions of plasma double layers and solitary waves (e.g., Block, 1972; Swift,
While these steady-state solutions show that such structures can exist, they do not describe how they form. The time evolution of these structures has been studied through particle-in-cell (PIC) simulations (e.g., Goertz and Joyce, 1975; Sato and Okuda, 1981; Barnes et al., 1985). Since these PIC simulations generally model a limited region of space, the results depend on which boundary conditions are imposed on the simulation box. Such localized models, while essential for understanding the detailed behavior within the region of parallel electric fields, avoid the important question of how the energy that accelerates the auroral particles is provided. A global scale electrostatic model of the auroral field line has been presented by Schriver (1999). This model is driven by the injection of ion fluxes at the upper end of the system; however, since this simulation is electrostatic, the electromagnetic energy input is not included.

In contrast to these kinetic models of auroral acceleration, the time development of auroral currents and fields has often been considered in terms of Alfvén wave propagation (Mallinckrodt and Carlson, 1978; Goertz and Boswell, 1979; Lysak and Dum, 1983; Seyler, 1988). While the Alfvén wave does not carry a parallel electric field that can accelerate auroral particles according to ideal MHD, a parallel electric field does develop when the spatial scale size perpendicular to the background magnetic field becomes comparable to the ion acoustic gyroradius (Hasegawa, 1976) or the electron inertial length (Goertz and Boswell, 1979). At altitudes of less than about 2-3 R_E where much of the auroral acceleration is thought to occur, the electron inertial effect is most important (Lysak and Carlson, 1981). This has led to a variety of models that have considered the development of parallel electric fields due to electron inertia (e.g., Seyler, 1988; Lysak, 1993, 1998; Lysak and Song, 2000; Rönmark and Hamrin, 2000; Seyler and Wu, 2001). In particular, some models have invoked Alfvén field line resonances that collapse to the electron inertial scale to explain auroral arc formation (Rankin et al., 1993, 1994, 1999a,b; Streltsov and Loiko, 1995, 1996, 1999; Streltsov et al., 1998; Rankin and Tikhonchuk, 1998; Bhattacharjee et al., 1999). In addition to these global resonances, a resonant cavity can be formed by the rapid decrease of the plasma density above the ionosphere, which causes an increase in the Alfvén speed from less than 1000 km/s in the ionosphere to greater than 100,000 km/s in the acceleration region at a few thousand kilometers altitude. The resonant frequency of this cavity, which has been called the ionospheric Alfvén resonator (IAR) by Polyakov and Rapoport (1981; see also Trakhtengertz and Feldstein, 1981, 1984, 1987, 1991; Lysak, 1986, 1988, 1991, 1993), is in the 0.1-1.0 Hz range. This density gradient also implies that the region where the drift velocity corresponding to a given field-aligned current maximizes is also localized, as is the effect of the electron inertial term. It should be noted that waves in this frequency range have been observed in the auroral zone by Viking (Block and Fälthammar, 1990; Marklund et al., 1990), Freja (Louarn et al., 1994; Grzesiak, 2000), and FAST (Sigsbee et al., 1998; Chaston et al., 1999), and at higher altitudes by Geotail (Sigsbee et al., 1998).

Although Alfvén wave models have been very useful in describing the evolution of auroral fields and currents, fluid models of Alfvén waves are inadequate to self-consistently describe particle acceleration due to the fields of these waves. One approach to model the particle acceleration due to Alfvén waves has involved PIC simulations that take Alfvén wave dynamics into account (e.g., Otani et al., 1991; Goertz et al., 1991; Hui and Seyler, 1992; Silberstein and Otani, 1994; Clark and Seyler, 1999; Génot et al., 1999, 2000, 2001a,b). These works have had some success at modeling features of auroral particle acceleration, but are still limited to small scales, so that they do not reflect the large-scale resonant structures described above. Large scale modeling of electron acceleration by Alfvén waves has been considered by
test particle models (Kletzing, 1994; Thompson and Lysak, 1996; Chaston et al., 1999, 2000, 2002; Kletzing and Hu, 2001). This work, in particular that of Chaston et al. (2000, 2002) and Kletzing and Hu (2001), has shown that the dispersed electron signatures frequently observed in field-aligned electron bursts can be reproduced by the Alfvén wave model. At higher altitude, electron acceleration has been associated with observations of small-scale Alfvén waves at the plasma sheet boundary layer (Wygant et al., 2002) and has been shown to be consistent with the kinetic Alfvén wave dispersion relation in the regime where electron pressure rather than electron inertia supports the parallel electric field (Lysak and Lotko, 1996; Lysak, 1998).

These models based on the electron inertial term treat the entire electron population as cold, neglecting the fact that hot magnetospheric plasma exists on auroral field lines, and indeed, can be the dominant component of the plasma in the auroral acceleration region (e.g., Strangeway et al., 1998). In an effort to include the kinetic effects of hot electrons, Rankin et al. (1999b) and Tikhonchuk and Rankin (2000) have developed a model in the context of field line resonances that includes a kinetic description of electrons bouncing between mirror points in the two hemispheres. While this model is a first step toward including electron kinetic effects into an Alfvén wave model, it is restricted to field line resonance cases and only considers one of the electron populations that exist on auroral field lines.

The purpose of this paper is to apply this kinetic approach to Alfvén waves that propagate along the plasma sheet boundary layer, as observed by Polar (Wygant et al., 2000, 2002; Keiling et al., 2000), and their extension into the ionosphere, where they are influenced by the ionospheric Alfvén resonator. At high altitudes, the plasma parameters vary slowly, and a local approach can be used to investigate the kinetic effects, as in previous work (Lysak and Lotko, 1996; Lysak, 1998). Near the ionosphere, however, the plasma parameters vary rapidly, and the electron kinetics must be treated non-locally. While Rankin et al. (1999b) and Tikhonchuk and Rankin (2000) concentrated on the effects of mirroring particles, the present paper will concentrate on the kinetics of passing electrons that propagate from the magnetosphere to the ionosphere, as well as ionospheric particles that propagate into the magnetosphere. In the remainder of this paper, we will first review the theory of kinetic Alfvén waves in a uniform plasma, concentrating on the effects on the electron distribution. Then the kinetic theory of Alfvén waves in the ionospheric Alfvén resonator will be presented, based on a simplified model that neglects the dipole magnetic geometry as in Lysak (1991). Although this model does not include all aspects that are important in this problem, it does give a simplified framework in which the kinetic theory can be understood. The paper will conclude with a discussion of the results and an outline of future work to extend the results presented here.

Local Kinetic Theory of Alfvén Waves

Recently, Polar observations of strong Alfvénic turbulence in the plasma sheet boundary layer (PSBL) have drawn renewed attention to Alfvénic acceleration of auroral electrons. Wygant et al. (2000) noted that large amplitude (∼ 100 mV/m) Alfvén waves are seen by Polar in the PSBL at 4-6 Re, on field lines that are conjugate to the aurora as seen by the UVI instrument. The Poynting flux estimated in these waves is the order of 1 mW/m² at the spacecraft, which maps to over 100 mW/m² at the ionosphere. The waves are seen most often when Polar crosses the PSBL during the expansion phase of substorms (Keiling et al., 2000, 2001), and expand into the central plasma sheet during large storms (J. R. Wygant, personal communication).

In addition to these larger-scale waves, there is also a considerable amount of fine
structure in the Alfvén waves, and these smaller-scale waves have a larger ratio of the wave electric to magnetic fields. This enhanced ratio could indicate that the waves have been modified by kinetic effects (Lysak, 1998), or that waves reflected from an insulating ionosphere or from the auroral acceleration region (Vogt and Haerendel, 1998) are present. The absence of any significant phase shift between the electric and magnetic fields and the lack of reflected Poynting flux suggest that kinetic effects are the most important influence on this ratio. Wygant et al. (2002) used this ratio and the measured plasma parameters together with the theoretical results of Lysak (1998) to find that the perpendicular wavelength of these waves was comparable to the electron inertial length. The theory was then used to estimate the ratio of the parallel to perpendicular potentials in the wave, and the estimated parallel potential was consistent with the energy of field-aligned electrons measured by the Hydra instrument on Polar. Angelopoulos et al. (2002) compared the Polar observations with those at Geotail at 18 RE, and found that the Poynting flux was much larger at Geotail, indicating that dissipation of the wave energy had taken place between the two satellites.

These observations suggest that an evaluation of energy flow and dissipation of kinetic Alfvén waves is in order. Such an evaluation should start with an analysis of the local dispersion relation, which can be written in matrix form as (Lysak and Lotko, 1996):

\[
\begin{pmatrix}
\varepsilon_{xx} - n_{e}^2 & n_{e} n_{x} \\
 n_{e} n_{x} & \varepsilon_{zz} - n_{e}^2
\end{pmatrix}
\begin{pmatrix}
E_{x} \\
E_{z}
\end{pmatrix} = 0
\]  

(1)

where the elements of the dielectric tensor are given by:

\[
\varepsilon_{xx} = 1 + \frac{c^2}{V_A^2} \frac{1 - \Gamma_0(\mu_i)}{\mu_i}
\]

(2)

\[
\varepsilon_{zz} = 1 + \frac{\Gamma_0(\mu_e)}{k \lambda_{De}^2} (1 + \chi Z(\chi))
\]

(3)

where \(\mu_{i,e} = k^2 \rho_{i,e}^2\), \(\chi = \omega/k_a\), \(\rho_{i,e}\) is the thermal ion (electron) gyroradius, \(a_e = (2T_e/m_e)^{1/2}\) is the electron thermal speed, \(\lambda_{De} = (e^2 T_e/m_e)^{1/2}\) is the Debye length, \(Z(\chi)\) is the plasma dispersion function and \(\Gamma_0(\mu) = e^{-\mu} I_0(\mu)\) is the modified Bessel function. As usual, the dispersion relation is found by taking the determinant of the matrix in equation (1). For the parameters of the Polar observations, we have \(\chi \ll 1\) and \(V_A \ll c\); thus, expanding the \(Z\) function and using the Padé approximation for the Bessel function (e.g., Johnson and Cheng, 1997), the dispersion relation can be written as:

\[
\omega^2 \approx k_z^2 V_A^2 \left[1 + k_z^2 \rho_i^2 + k_z^2 \rho_e^2 \left(1 - i\sqrt{\pi \chi}\right)\right]
\]

(4)

where \(\rho_{i,e}^2 = T_i m_i / e^2 B^2\) is the ion acoustic gyroradius. Note the imaginary term that gives the Landau damping of the wave. Expanding the \(Z\) function in the cold electron limit, \(\chi \gg 1\), gives the inertial limit of the Alfvén wave dispersion relation, including the effects of hot ions

\[
\omega^2 \approx k_z^2 V_A^2 \frac{1 + k_z^2 \rho_i^2}{1 + k_z^2 \lambda_e^2}
\]

(5)

where \(\lambda_e\) is the electron inertial length. It should be noted that there is essentially no Landau damping in the cold plasma limit since the wave phase velocity is on the tail of the electron distribution.
The polarization relations between the components of $E$ and $B$ can be found by solving the full equation (1) and by using Faraday’s law (Lysak, 1998). In the warm electron limit, these relations become:

$$\frac{E_z}{E_x} \approx - \frac{k_x \rho_s^2}{1 + k_x^2 \rho_s^2} \left(1 - i \sqrt{\pi} \chi\right)$$  \hspace{1cm} (6)

$$\frac{E_x}{B_y} \approx V_A \frac{1 + k_x^2 \rho_i^2}{\sqrt{1 + k_x^2 \rho_s^2 + k_x^2 \rho_i^2}}$$  \hspace{1cm} (7)

while for cold electrons, the wave components are related by

$$\frac{E_z}{E_x} \approx \frac{k_x \lambda_c^2}{1 + k_x^2 \lambda_c^2}$$  \hspace{1cm} (8)

$$\frac{E_x}{B_y} \approx V_A \sqrt{\left(1 + k_x^2 \rho_i^2\right)\left(1 + k_x^2 \lambda_c^2\right)}$$  \hspace{1cm} (9)

Note that the parallel electric field $E_z$ is produced by the electron pressure term in the warm electron limit, through the ion acoustic gyroradius $\rho_s$, or by the effect of electron inertia for cold electrons, and not, as is sometimes stated, by ion gyroradius effects. Indeed, the ion gyroradius effect appears in the denominator of equation (6), indicating that hotter ions tend to reduce the parallel electric field in the warm plasma case. Finite ion gyroradius does not affect the parallel electric field in the cold electron limit, as is seen in equation (8). It should also be noted that the sign of the parallel electric field reverses between the warm electron limit, equation (6), and the cold electron limit, equation (8). Equations (7) and (9) demonstrate that the ion gyroradius effect increases the $E_x/B_y$ ratio, as was indicated by the Polar observations. This ratio also has a small imaginary part in the warm plasma case; however, the full numerical solution indicates that it is negligible except when the wave is strongly damped. It should be noted that equations (4)-(9) give the limiting cases of these quantities in the warm and cold plasma limits; these ratios are plotted without these approximations in Lysak and Lotko (1996) and Lysak (1998).

We wish to consider the energy flow and dissipation produced by these waves. The starting point for such a calculation is Poynting’s theorem for the wave energy averaged over a wave period, which we can write in the form

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{S} = -P$$  \hspace{1cm} (10)

where the wave energy, Poynting flux, and power dissipation are defined by

$$W = \frac{\text{Re} \left( \mathbf{E}^* \cdot \mathbf{E} \right)}{4} + \frac{\left| \mathbf{B} \right|^2}{4 \mu_0} \quad \mathbf{S} = \frac{1}{2 \mu_0} \text{Re} \left( \mathbf{E}^* \times \mathbf{B} \right) \quad P = \frac{1}{2} \text{Re} \left( \mathbf{j}^* \cdot \mathbf{E} \right)$$  \hspace{1cm} (11)

We consider a situation in which we assume that a wave with frequency $\omega$ and perpendicular wave number $k_z$ is emitted from a source at $z = 0$. Then we consider the propagation of the wave away from this source. Since this process is steady-state after averaging over the wave period, the time derivative term in equation (10) goes to zero. Thus, in this scenario, the wave Poynting flux decays away from the source due to the Landau dissipation $P$. To calculate the Poynting flux and the dissipation, the dispersion relation given by equation (1) is solved for the complex parallel wave number $k_z$, and equation (1) together with Maxwell’s equations can be used to
calculate the Poynting flux and power dissipation associated with the wave. It should be noted that the non-vanishing wave field components in this case are $E_x$, $B_y$, and $E_z$.

Although the Poynting flux of the ideal MHD Alfvén wave is strictly along the magnetic field, the kinetic effects cause the wave energy to move across field lines as well. The parallel component of the Poynting flux is

$$S_z = \frac{1}{2\mu_0} \Re E_x^* B_y = \frac{|E_x|^2}{2\mu_0 V_A} \Re \frac{V_A B_y}{E_x}$$

while the perpendicular Poynting flux is given by:

$$S_x = -\frac{1}{2\mu_0} \Re E_z^* B_y = -\frac{|E_z|^2}{2\mu_0 V_A k_x V_A} \Re \left[ \left( \frac{k_x V_A}{\omega} \right) \left( \frac{V_A B_y}{E_x} \right)^* \left( \frac{k_x E_z}{k_x E_y} \right) \right]$$

Comparing equations (12) and (13), we can relate the two components of the Poynting flux as

$$S_x = -\left\{ \frac{\Re \left[ \left( \frac{k_x V_A}{\omega} \right) \left( \frac{V_A B_y}{E_x} \right)^* \left( \frac{k_x E_z}{k_x E_y} \right) \right]}{\Re \left( \frac{V_A B_y}{E_x} \right)} \right\} \frac{\omega}{k_x V_A} S_z$$

(14)

As noted above, the imaginary part of $B_y/E_x$ is very small, so the perpendicular Poynting flux is closely related to the real part of the scaled parallel electric field. Calculation of the dimensionless quantity in braces in equation (14) shows that this quantity is less than 0.3. The quantity $\omega/k_x V_A = \lambda_x / V_A \tau$, where $\lambda_x$ is the perpendicular wavelength and $\tau$ is the wave period.

The Polar observations (Wygant et al., 2002) indicate that the perpendicular wavelength is the order of 10 km, while a typical value of the Alfvén speed in the plasma sheet boundary layer is 1000 km/s and wave periods may be estimated to be the order of 1 minute. This suggests that $\omega/k_x V_A < 10^{-3}$; therefore, the deviation of the Poynting flux from the parallel direction is quite small for these parameters.

The energy dissipation can be determined by looking at the work done on the electrons by the wave:

$$P = \frac{1}{2} \Re \left( j_z E_z \right) = \frac{1}{2\mu_0} \Re \left( -ik_x B_y^* E_z \right) = \frac{|E_z|^2}{2\mu_0 V_A} \Im \left[ \left( \frac{k_x V_A}{\omega} \right) \left( \frac{V_A B_y}{E_x} \right)^* \left( \frac{k_x E_z}{k_x E_y} \right) \right]$$

(15)

Note that only the parallel current and electric field enter into this expression since the polarization current is out of phase with the perpendicular electric field. Again, it is useful to relate this to the parallel Poynting flux, so we can write

$$P = \left\{ \frac{\Im \left[ \left( \frac{k_x V_A}{\omega} \right) \left( \frac{V_A B_y}{E_x} \right)^* \left( \frac{k_x E_z}{k_x E_y} \right) \right]}{\Re \left( \frac{V_A B_y}{E_x} \right)} \right\} \frac{\omega}{V_A} S_z$$

(16)

This power dissipation is related to the imaginary part of the scaled parallel electric field, which is directly related to the Landau damping, as can be seen from equations (4) and (6). Note that this scenario gives a characteristic parallel scale length of $V_A/\omega = V_A \tau / 2\pi$. For $V_A = 1000$ km/s and a wave period of 60 s, this scale length is 10,000 km.
equation (16) in the same format as was used in Lysak and Lotko (1996) and Lysak (1998) as a function of the parameters \( v_e^2 / V_A^2 = T_e / m_e V_A^2 \) and \( k_c \omega / \omega_{pe} \). Since the plasma sheet ions are generally somewhat hotter than the electrons, this plot shows results for temperature ratios \( T_i / T_e = 1, 2, 5, \text{ and } 10 \). Since the dissipation rate is proportional to the Poynting flux and since the perpendicular Poynting flux is negligible equation (10) in the steady state can be written as

\[
\frac{\partial S_z}{\partial z} = -P = -2\kappa S_z
\]

This equation indicates that the Poynting flux decays spatially as \( S_z(z) = S_z(0)e^{-2\kappa z} \). Thus, \( \kappa \) can be interpreted as a spatial damping rate of the wave, with the factor of 2 being present since the Poynting flux is a quadratic quantity. This decay of the Poynting flux is due to the conversion of the electromagnetic energy to the energy of the Landau-resonant electrons.

According to the results of Wygant et al. (2002), the Polar observations occur in a regime of this plot where \( v_e^2 / V_A^2 = T_e / m_e V_A^2 \) is between 3 and 30. The observed ratio of the electric field to the magnetic electric field is in the range 2-10, indicating that \( k_c \omega / \omega_{pe} \) ranges from 0.5 to 3 if \( T_i / T_e = 1 \) and 0.1 to 1 for a temperature ratio of 10. For these parameters, it can be seen that the normalized power dissipation is about 0.1 for equal temperatures and 0.01 for hotter ions. Thus, the local heating of electrons is favored by cooler ion temperatures. For the canonical numbers of 1000 km/s for the Alfvén speed and 1 minute for the wave period, this indicates that the decay scale length \( L = 1/2\kappa \sim 15 \text{ RE} \) for the equal temperature case, and up to 10 times larger for \( T_i / T_e = 10 \). Over this distance, 64\% of the wave Poynting flux is converted to electron energy. This would indicate that 10\% of the wave Poynting flux is converted to particle energy over a distance of 1.5-15 RE. Since \( \kappa \) scales as \( 1 / V_A \tau \), the efficiency of the energy conversion is higher for lower Alfvén speeds and shorter wave periods.

A difficulty in these considerations is that it is difficult to determine the actual frequency of the wave. The time variation of the fields measured by Polar is most likely due to the Doppler shift due to the relative motion of the spacecraft and plasma sheet. Theoretical considerations combined with the ratio of the perpendicular electric and magnetic perturbations can be used to estimate the perpendicular wavelength, as in Wygant et al. (2002); however, only the parallel phase velocity, i.e., \( \omega / k_z \) can be determined from the theory and available measurements, but not \( \omega \) or \( k_z \) separately.

This ambiguity also leads to uncertainty in the applicability of the local model to the Polar measurements. The local approximation requires that the wavelength of the wave be smaller than the scale of variations in the Alfvén speed along the auroral field line. Thus, low frequency global modes, such as those considered by Rankin et al. (1999b), must be treated with a non-local model. However, higher frequency waves, such as the 0.9 Hz waves seen on Geotail by Sigsbee et al. (1998), will be well described by the local model. Generally, in the tail where the typical scales along field lines may be many Earth radii and the Alfvén speeds are less than 1000 km/s, waves with frequencies above 10 mHz (wave periods less than 100 s) can be described by the local approximation. The Polar measurements are in the transition region where the local approximation is beginning to break down. In this inner magnetospheric region, the local theory must be replaced by a non-local theory that takes into account the mode structure along field lines. A first step toward such a model will be described in the next section.
Non-local kinetic theory of the ionospheric Alfvén resonator

As noted above, when the gradients in the plasma parameters become comparable to a wavelength, the local theory described above is not valid. Use of such a non-local theory is particularly important when one considers the kinetic theory of fundamental field line resonances (e.g., Rankin et al., 1999b; Tikhonchuk and Rankin, 2000), or when treating modes of the ionospheric Alfvén resonator, as will be presented here. These modes intrinsically have variations on the same scale as the inhomogeneities of the plasma. In such cases, one must directly integrate the Vlasov equation, taking into account the inhomogeneous structure of the plasma. Integration over velocities can then lead to an integral equation giving the field-aligned current as an integral over the parallel electric field. If the field-aligned current is known, this integral equation must be solved for the parallel electric field. This procedure is outlined in the Appendix.

Carrying out this procedure is in general quite complex, and all of the different possible particle orbits should be included. Tikhonchuk and Rankin (2000), for example, have included trapped orbits in a dipolar magnetic field. For our studies of waves on field lines that connect to the PSBL, this formulation may not be appropriate since the field lines may be open, or long enough so that the particles may scatter before they mirror. In addition, the presence of an equilibrium parallel electric field leads to the presence of multiple particle populations that can be reflected by the parallel electric field or trapped between the magnetic mirror and the electrostatic mirror, as in the calculations of Chiu and Schulz (1978), Chiu and Cornwall (1980), and Ergun et al. (2000).

To avoid these complications while still obtaining new information about the kinetic effects, we will adopt the simplified model of the ionospheric Alfvén resonator on straight magnetic field lines, as in the work of Lysak (1991). We will calculate the wave fields and the field-aligned current according to this model, and then solve the Vlasov equation to determine the electron kinetic response. This background model assumes long perpendicular wavelengths so that to lowest order the parallel electric field may be neglected. The effect of the kinetic particles will then be treated as a perturbation on this background. In this approximation, the wave equations can be determined from Faraday’s Law and Ampere’s Law including the perpendicular polarization current. Assuming perturbations at a frequency \( \omega \), these equations become

\[
\frac{\partial E_z}{\partial z} = i \omega B_y, \quad \frac{\partial B_y}{\partial z} = i \omega \varepsilon_{xx} \mu_0 E_x \approx \frac{i \omega}{V^2_A(z)} E_x
\]

where the perpendicular dielectric tensor component \( \varepsilon_{xx} \) is given by equation (2). Note that the approximate form in the second equation holds in the long wavelength limit, where the Bessel function factors in equation (2) reduce to unity, and for \( V^2_A \ll c^2 \), which does not always hold on auroral field lines. When this inequality is not valid, the Alfvén speed should be replaced by \( V^2_A / (1 + V^2_A / c^2) \) in equation (18).

The plasma density in this model is given by

\[
n(z) = n_i \left( e^{-z/h} + \varepsilon^2 \right)
\]

(19)

Note that the scale height \( h \) is typically a few hundred kilometers in the auroral zone. In this case the Alfvén velocity and the electron inertial length \( \lambda_e \) can be written as

\[
V^2_A(z) = \frac{V^2_{Al}}{e^{-z/h} + \varepsilon^2}, \quad \frac{\lambda^2_e}{e^{-z/h} + \varepsilon^2}
\]

(20)
where $V_{Al}$ and $\lambda_{ei}$ are the Alfvén velocity and inertial length, respectively, at the ionosphere. The solutions to the wave equations (18) with the Alfvén speed profile (20) can be written in terms of Bessel functions. There are two solutions, one corresponding to a wave with downward Poynting flux, and the other with upward Poynting flux. Let us consider the downward-propagating wave, which has the solution (Lysak, 1991)

$$\delta E_x = -ik\Phi_0 J_{\xi_0 e} \left(\xi_0 e^{-z/2h}\right)$$

(21)

where $\xi_0 = 2h\omega/V_{Al}$ and the amplitude of the wave is given in terms of the perpendicular potential drop $\Phi_0$. Now the magnetic perturbation can be written as

$$\delta B_y = -\frac{i}{\omega} \frac{\partial \delta E_x}{\partial z} = -\frac{k_s \Phi_0}{V_{Al} \xi_0} J'_{\xi_0 e} (\xi)$$

(22)

where the prime on the Bessel function indicates differentiation with respect to $\xi = \xi_0 e^{-z/2h}$. Note that these expressions are valid in the limit $k_s^2 \lambda_{M}^2 \ll 1$; for general values of this parameter, a numerical solution of the eigenmode equations must be obtained as in Lysak (1993). Now using Ampère’s Law, the field-aligned current becomes

$$\delta j_z = \frac{ik_s}{\mu_0} \delta B_y = \frac{ik_s \Phi_0}{\mu_0 V_{Al} \xi_0} J'_{\xi_0 e} (\xi)$$

(23)

If that cold electrons carry the field-aligned current, the electron equation of motion gives the parallel electric field

$$\delta E_{z,cold} = \frac{m_e}{n e^2} (\omega \delta j_z) = \frac{\Phi_0}{2h} k_s^2 \lambda_{M}^2 \xi_0 e^{-z/2h}$$

(24)

This cold plasma expression will be used for comparison with the kinetic expression below.

Now we can evaluate the perturbation in the distribution function. We will neglect any background parallel electric fields, so that the unperturbed parallel velocity is constant, and so we can write the travel times as $\tau(z_1, z_2) = (z_2 - z_1)/v_z$. First, consider the upgoing particles, where the perturbed distribution function is given by equation (A8). These particles leave the ionosphere before interacting with the wave, so that we can take $\delta f_+(0) = 0$. Thus, the upgoing distribution function can be written as

$$\delta f_+ (z) = -q \frac{\partial f_{0+}}{\partial W} \int_0^z dz' \delta E_z (z') e^{i\omega(z-z')/v_z}$$

(25)

The solution for the downgoing particles is more complicated. Strictly speaking, there is no upper boundary $z = L$, since the model extends to infinity. However, at high enough altitudes, where $e^{-z/h} \ll \xi^2$, the solution becomes a plane wave solution (Lysak, 1991). Thus, at this location we can adopt the uniform plasma solution as described in the previous section. The perturbed distribution function in this case is given by

$$\delta f_- (L) = -\frac{iv_z}{\omega + i\eta - k_z v_z} q \frac{\partial f_0}{\partial W} \delta E_z (L)$$

(26)

where $k_z = -\omega/V_{Al}$ is the wave number of the wave in the uniform region as can be approximated by equation (4). Note that this wave number is negative since the wave is propagating in the $-z$ direction. The small positive imaginary number $\eta$ has been inserted to take into account the Landau prescription for resolving the resonance at $\omega = k_z v_z$. As usual the limit $\eta \to 0^+$ is implicit. Inserting equation (26) into the solution given by equation (A9) gives the
distribution for negative velocities
\[ \delta f_-(z) = q \frac{\partial f_0}{\partial W} \left[ -\frac{iv}{\omega + m - k_v v} e^{-i\alpha (L+z)/v} \delta E_z(L) + \int dz' \delta E_z(z') e^{-i\alpha (z-z')/v} \right] \] (27)

In order to compare these results with those from the local theory, we need to calculate the current carried by these hot particles, as described in the Appendix. Inserting equations (25) and (27) into equation (A11), we find
\[ \delta j_z = -q^2 \int_0^\infty dv \nu \left[ \frac{\partial f_0}{\partial W} \int_0^\infty dz' e^{i\alpha (z-z')/v} \delta E_z(z') + \frac{\partial f_0}{\partial W} \int_0^L dz' e^{i\alpha (z-z')/v} \delta E_z(z') \right. 
\left. - \frac{i}{\omega} \frac{vV_M}{\partial W} \frac{1}{v-V_M(1+i\eta/\omega)} e^{i\alpha (L-z)/v} \delta E_z(L) \right] \] (28)

Note that this expression simplifies if the upgoing and downgoing distribution functions are the same
\[ \delta j_z = -q^2 \int_0^\infty dv \nu \left[ \frac{\partial f_0}{\partial W} \int_0^\infty dz' e^{i\alpha (z-z')/v} \right. 
\left. \int_0^\infty dv \nu \frac{1}{v-V_M(1+i\eta/\omega)} e^{i\alpha (L-z)/v} \delta E_z(L) \right] \] (29)

Choosing a form for the distribution function, we can evaluate these integrals. We take a Maxwellian for the electron distribution,
\[ f_0 = n_0 \sqrt{\frac{m_e}{2\pi T_e}} e^{-m_v^2/2T_e} = n_0 \sqrt{\frac{m_e}{2\pi T_e}} e^{-W/T_e} \] (30)

It can be seen that \( \partial f_0 / \partial W = -f_0 / T_e \). Then the first of the two integrals in equation (29) becomes
\[ \delta j_1(z) = \frac{n_0 q^2}{m_e} \int_0^L dz' \delta E_z(z') \sqrt{\frac{m_e}{2\pi T_e}} \int_0^\infty dv \nu \frac{1}{v} e^{-m_v^2/2T_e} e^{i\alpha (z-z')/v} \] (31)

Note that this integral can be written in the form of equation (A12)
\[ \delta j_1(z) = \int_0^L dz' \sigma(z,z') \delta E_z(z') \] (32)

where the conductivity kernel is given by
\[ \sigma(z,z') = \frac{i n_0 q^2}{m_e \omega} \left[ -\frac{2i\omega}{\sqrt{\pi a_e}} \int_0^\infty d\zeta \zeta e^{-\zeta^2} e^{i\alpha (z-z')/a_e \zeta} \right] \] (33)

where the thermal velocity \( a_e = (2T_e/m_e)^{1/2} \) and the integral has been written in terms of the dimensionless variable \( \zeta = v/a_e \). Note that the factor in front of the brackets is simply the conductivity for the cold plasma case. The dimensionless integral in equation (33) is plotted in Figure 2 as a function of the parameter \( \omega |z-z'|/a_e \). It can be seen that the main contribution to this integral occurs when \( |z-z'| \leq 2\pi a_e / \omega \), i.e., roughly the distance a thermal particle travels in one wave period. It can also be verified that the bracketed term integrated over the spatial coordinates is 1. Thus, in the limit of a cold plasma, the bracketed term becomes a delta-function.
and the cold plasma conductivity is recovered.

Turning to the boundary value integral represented by the second line of equation (29), similar manipulations can cast this in the form

\[
\delta j_z(z) = \frac{i n_0 e^2}{m_e \omega} \sqrt{\frac{m_e}{2 \pi T_e}} e^{-m_e v^2/2T_e} \left[ v - \frac{v^2}{v-V_M(1+i\eta/\omega)} \right] \delta E_z(L) e^{i\omega(L-z)/v}
\]

\[
\equiv \sigma_L(z, L) \delta E_z(L)
\]

This expression, evaluated at \( z = L \), would give the local, kinetic response for the plasma if the integral were over all velocities; however, in this case, only the downgoing particles are included. It can be verified that the first term in the square brackets gives the local, cold plasma response. It can also be seen that when the resonant velocity is much greater than the particle velocity, the second term is smaller by the ratio \( v/V_M \).

As an example of the response of the hot plasma to Alfvén waves in the ionospheric resonator, consider a model in which \( \varepsilon = 0.1 \), i.e., the Alfvén speed is 10 times larger in the magnetosphere than in the ionosphere. The model Alfvén speed profile for this case is shown in Figure 3. If we assume a typical value for the scale height of 600 km (0.1 R_E), the range of this calculation goes roughly to 3 R_E geocentric distance. The cold ionospheric density will be given by the exponential term in equation (19), while the kinetic particles will have a density \( n_{0i} = \varepsilon n_i \). We will take the density and temperature of the upgoing and downgoing kinetic particles to be the same, although in general they need not be. We will assume that the field-aligned current is given by the Alfvén resonator model given by equation (23). We note that while this model is certainly not an exact description of the auroral field line, it contains the necessary features to illustrate the physics of the electron kinetic effects on a realistic field line. A generalization of this model to a more realistic situation will be the subject of future work.

Note that the cold ionospheric particles have a response given by the local conductivity, \( \sigma_{\text{cold}} = i n_{\text{cold}}(z)e^2 / m_e \omega \). Then the parallel electric field is given by the solution of the integral equation

\[
\delta j_z(z) = \sigma_{\text{cold}}(z) \delta E_z(z) + \int_0^L dz' \sigma(z, z') \delta E_z(z') + \sigma_L(z, L) \delta E_z(L)
\]

At high altitudes, the solution to this equation must become equal to the kinetic solution of the previous section, which can be derived from equation (3)

\[
\delta j_z = -2i n_0 e^2 \chi^2 \left( 1 + \chi Z(\chi) \right) \delta E_z \equiv \sigma_{\text{kin}} \delta E_z
\]

where \( \chi = \omega / k_{\parallel} a_e \) is found by solving the dispersion relation given by equation (1) using the magnetospheric parameters. At low altitudes, the conductivity is given by the cold plasma response in the ionosphere. Thus the first trial solution can be written as

\[
\delta E_0(z) = \frac{\delta j_z(z)}{\sigma_{\text{cold}}(z) + \sigma_{\text{kin}}}
\]

Note that the kinetic conductivity is smaller by a factor \( \varepsilon^2 \) at low altitudes, but dominates at high altitudes where the cold density goes to zero. Thus, this trial solution satisfies the boundary conditions for this problem.

This trial solution itself is only an approximate solution to equation (35), and so it is supplemented by a series of Bessel functions that constitute a complete set for functions that
vanish at \( z = 0 \) and as \( z \to \infty \) (e.g., Jackson, 1975, section 3.7). Thus the full trial solution is

\[
\delta E_z(z) = c_0 \delta E_0(z) + \sum_{i=1}^{N} c_i J_1(\eta_i e^{-z/2h})
\]

(38)

Here \( \eta_i \) denotes the \( i \)th zero of \( J_1(x) \). A least-squares minimization procedure is used to calculate the set of \( c_i \) that gives the best solution to equation (35).

We will present the results of this calculation in terms of an effective conductivity given by the current divided by the local value of the parallel electric field, normalized to the cold plasma conductivity

\[
\sigma_{\text{eff}}(z) = \frac{\delta j_z(z)}{\delta E_z(z)} = \frac{i n(z) e^2}{m_e \omega} \hat{\sigma}_{\text{eff}}(z)
\]

(39)

Figure 4 plots the value of this dimensionless conductivity \( \hat{\sigma}_{\text{eff}} \) for four values of the thermal speed of the hot particles, \( a_e = (2T_e / m_e)^{1/2} \), normalized by the Alfvén speed in the ionosphere. The real part of the effective conductivity is plotted as a solid line, while the dashed line gives the imaginary part. A value of \( k_e^2 \lambda^2 \) = 0.01 has been used in these plots. Note that this value corresponds to the left boundary in Figure 1. Figure 4 shows results for \( a_e / V_{Al} = 1, 3, 10, \) and \( 30 \), corresponding to \( v_e^2 / V_{Al}^2 = 0.005 \) to 4.5 in Figure 1. (Recall that the \( v_e \) in Figure 1 are defined without a factor of 2 in the thermal speed.) It should be noted that a typical Alfvén speed in the ionosphere is about 600 km/s, which is the thermal velocity for a 1 eV electron. Thus, these four plots roughly correspond to energies of 1, 10, 100, and 1000 eV. It can be seen in panel (a) that for \( a_e / V_{Al} = 1 \), the effective conductivity is nearly unity, indicating that the plasma response is essentially that of a cold plasma. Panel (b) shows that the real part of the conductivity in enhanced somewhat by thermal effects, while panels (c) and (d) show that the real part is reduced when the thermal speed is comparable or greater than the magnetospheric Alfvén speed. Note that the real part of the normalized conductivity refers to the inductive part of the parallel electric field, as in the electron inertial effect; thus, values of the real part less than 1 indicate regions where the kinetic effects will lead to a localized enhancement of the inductive parallel electric field.

The dashed lines in Figure 4 give the imaginary part of the normalized conductivity, which corresponds to a contribution to the field-aligned current that is in phase with the parallel electric field. It is this component that contributes to the dissipation of the wave. The total power dissipation averaged over a wave cycle can be given by

\[
P = \frac{1}{2} \text{Re} \left( \delta j_z \delta E_z^* \right) = \frac{1}{2} \text{Re} \sigma_{\text{eff}} |\delta E_z|^2 = -\frac{1}{2} \frac{n(z) e^2}{m_e \omega} \text{Im} \hat{\sigma}_{\text{eff}} |\delta E_z|^2
\]

(40)

This dissipation accelerates the electrons along the field line, and provides damping to the wave. Figure 5 plots this dissipation normalized to the incident Poynting flux as a function of the thermal speed for the runs shown in Figure 4, with \( a_e / V_{Al} = 1, 3, 10, \) and \( 30 \) corresponding to the solid, dotted, dashed, and dash-dot lines, respectively. For higher values of \( z \), the dissipation becomes uniform and is significant for the larger values of the thermal speed; thus, this damping is the extension of the Landau damping discussed in the previous section. In addition to this uniform Landau damping, there is also dissipation for \( z/2h \) between 3 and 5, in the transition region where the Alfvén speed gradient is significant. Note that for the hottest case, there is a region in which the dissipation is negative. In this region the local current and electric field are in opposite directions locally and the electrons lose energy. The total integrated dissipation
relative to the input Poynting flux is 0, 0.04%, 3.0%, and 7.7% for the four cases. This increase is due to the increase of the parallel electric field due to the thermal effects, since these calculations are taken under conditions of equal current. It should also be noted that increasing the perpendicular wave number leads to an increase in this damping; however, in this case the simplifying assumptions made here that the kinetic effects do not modify the wave structure are no longer justified.

Discussion

The model described above gives a preliminary calculation of hot plasma effects on the ionospheric Alfvén resonator. It should be noted that the typical value of the Alfvén speed in the ionosphere is comparable to the thermal speed of a 1 eV electron (600 km/s). Thus, a plasma sheet temperature of 100 eV-1 keV corresponds to $a_e/V_{Al} \sim 10-30$ in Figures 4 and 5. The scale height $2h$ is typically in the 1000 km range, so the transition region at $z \sim 10h$ is at about 1 $R_E$ altitude. Note that we have also made the assumption that the presence of the hot particles has not modified the mode structure or the spatial profile of the parallel electric field. This is in the spirit of plasma wave calculations that are done in the limit of weak damping or growth (e.g., Krall and Trivelpiece, 1973, section 8.6) in which the fluid equations are used to find the real part of the dispersion relation while the full kinetic description is used to calculate growth or damping. A more complete treatment, which will be left for future work, would be to consider the modifications of the mode structure caused by the kinetic effects.

Another significant simplification in the present calculations is the use of a straight magnetic field line model. The presence of the dipole magnetic geometry, and its extension into the plasma sheet for PSBL field lines, modifies the zero-order particle orbits that should be considered in equations (A3)-(A9). The magnetic mirror will lead to the reflection of hot incoming particles, as was considered in Tikhonchuk and Rankin (2000). In addition, the linearization of the Vlasov equation should be around a self-consistent equilibrium including a zero-order parallel electric field, such as that calculated by Ergun et al. (2000). An important feature that is introduced in such a model is the electrostatic reflection of ionospheric electrons by an upward parallel electric field. It is worth noting that the bounce time of superthermal electrons emitted from the ionosphere as backscattered primaries or secondary particles is comparable to the 1 second wave period of waves in the Alfvén resonator. Thus, it is likely that a bounce resonance feature would result. Such features are in fact seen in the test particle calculations of Thompson and Lysak (1996). Indeed, as the evolution of an auroral arc progresses, the density and temperature of such backscattered and secondary electrons might be expected to increase and the evolution of Alfvén waves would be modified.

It is worth noting that the type of electron acceleration produced here is distinct from the inverted-V monoenergetic electron beams that have been extensively studied. These calculations indicate that a region of heated electrons will be produced at altitudes at and above about 1 $R_E$ in altitude, and these heated electrons will propagate down the field lines. These electrons are heated primarily parallel to the magnetic field, and so they should overcome magnetic mirroring and precipitate into the ionosphere. It is worth noting that Polar Hydra data shows parallel heating of electrons in the region of large Alfvénic turbulence (Wygant et al., 2002). This event shows a bi-directional electron beam, which may be due to the non-resonant sloshing of electrons in the wave, along with a uni-directional population in the direction of wave propagation, suggestive of a Landau resonant tail. More coordinated wave-particle observations
of this type would be useful in determining the flow of energy from the plasma sheet into the auroral zone.

A final question that remains is the source of the waves produced at the plasma sheet boundary layer. If these waves are being dissipated by the acceleration of auroral electrons, they will be damped if not regenerated in some way. There are a number of possible mechanisms for producing these waves. First of all, since there are gradients perpendicular to the magnetic field in the PSBL, linear phase mixing will produce small perpendicular wavelength waves if large scale Alfvén waves are present in the boundary layer region. Such arguments have long been invoked to explain the narrow scales of field line resonances (e.g., Mann et al., 1995) and in the ionospheric Alfvén resonator (Lysak and Song, 2000). Shear Alfvén waves can be produced by linear mode conversion at the plasma sheet boundary, as in the original suggestion by Hasegawa (1976), who postulated that kinetic Alfvén waves can be produced from surface waves on the boundary. Variations on this model invoke mode conversion from plasma sheet waveguide modes (Allan and Wright, 1998) or from compressional waves in the plasma sheet (Lee et al., 2001). Nonlinear mode conversion can also occur during bursty reconnection (Song and Lysak, 1989b, 1994). Linking these waves to reconnection is an appealing idea since it is thought that the PSBL maps back to a distant neutral line. Such bursty reconnection in the presence of a By component in the lobe field would naturally emit shear Alfvén waves due to the conservation of magnetic helicity (Song and Lysak, 1989a; Wright and Berger, 1989).

Clearly, further study of the energy flows in the magnetotail and the dissipation of wave energy into the acceleration of auroral particles is necessary. The observations of Wygant et al. (2002) and Angelopoulos et al. (2002) have shown that a great deal of the energy flow in the tail is mediated by wave propagation through this region. Observations from Polar, Geotail and now Cluster should be coordinated to give a complete picture of these energy flows. Auroral satellites such as FAST as well as sounding rocket and ground observations can view the results of the dissipation of this wave energy into the acceleration of auroral particles. Thus, future observations should give further motivation to theoretical discussions of energy flow and dissipation in the tail.

In summary, recent observations as well as theoretical considerations presented here suggest that Alfvén waves can be efficient accelerators of auroral electrons. Such acceleration can be distinguished from the inverted-V electron acceleration in that it is confined in pitch angle but has a broad spectrum of energy. Landau dissipation both in the relatively uniform plasma sheet as well as its analog in the ionospheric Alfvén resonator can convert significant amounts of wave energy into the field-aligned acceleration of auroral electrons. While the calculations presented here only give a preliminary picture of these wave-particle interactions, they provide the basis for more sophisticated further studies of this interaction.

Appendix: Evaluation of the kinetic conductivity

To evaluate the non-local kinetic conductivity, we must integrate the Vlasov equation over unperturbed particle orbits. The first step in such a calculation is to calculate the perturbations in the electron distribution function, assuming that the profile of the parallel electric field perturbation is known. Then, this perturbation is integrated over all velocities to calculate the current. The result from this procedure will be an integral equation given the field-aligned current in terms of an integral over the parallel electric field, which can then be solved for the parallel electric field when the field-aligned current is given. The following procedure
follows a similar procedure outlined by Rankin et al. (1999b) and Tikhonchuk and Rankin (2000).

First, we must know the unperturbed particle orbits. For electrons interacting with low-frequency Alfvén waves, the magnetic moment of the electron orbit is an adiabatic invariant, and can be taken to be constant. A second constant of the motion can be taken to be the bounce action for periodic orbits, or equivalently, the total energy of the electron, given by

\[ W = \frac{1}{2} m v_z^2 + \mu B(z) + q\Phi(z) \]  

(A1)

where as usual \( \mu = m v_z^2 / 2B \) is the magnetic moment and \( \Phi \) is the electrostatic potential. Note that while the total energy is conserved for the unperturbed orbit, when Alfvén waves are present it can be changed by the parallel electric field of the wave. Thus, the first-order Vlasov equation in this case can be written as

\[ \left( \frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z} \right) \delta f_{\pm} (z, \mu, W, t) + \frac{dW}{dt} \frac{\partial f_{\pm} (\mu, W)}{\partial W} = 0 \]

(A2)

Here \( z \) represents the path length along the field line, and the background and perturbation distribution functions are separated into particles with positive parallel velocity ("+" subscript) and those with negative parallel velocity ("-" subscript). Here we will take positive velocity to be upward away from the ionosphere. Note that in general the unperturbed particle orbits may have turning points where they are reflected either by the magnetic gradient or by the background parallel electric field. At such points \( z_t \) the distributions must satisfy \( \delta f_{\pm}(z_t) = \delta f_{\pm}(-z_t) \). The energy change is given by the work done by the parallel electric field, \( dW / dt = q\delta E_z v_z \).

Equation (A2) can be solved by integrating over the unperturbed trajectory of a particle over past times

\[ \delta f_{\pm} (z, t) = \delta f_{\pm} (z_0, t_0) - q \int_{t_0}^{t} dt' \delta E_z (z', t') v_z (t') \frac{\partial f_{\pm} (\mu, W)}{\partial W} \]

(A3)

where the dependencies on the constants \( \mu \) and \( W \) are implicit. Here the orbit is defined by

\[ z(t) = z_0 + \int_{t_0}^{t} dt' v_z (t') \]

(A4)

and we have adopted the notation that \( z = z(t) \) and \( z' = z(t') \). However, it can be seen that the velocity can be defined as a function of position using equation (A1) and so it is more convenient to write equations (A3) and (A4) as functions of position. Thus, equation (A4) can be inverted to find

\[ t = t_0 + \int_{z_0}^{z} \frac{dz'}{v_z (z')} \equiv t_0 + \tau (z_0, z) \]

(A5)

where the last form defines the travel time \( \tau \). Then equation (A3) can be written as

\[ \delta f_{\pm} (z, t) = \delta f_{\pm} (z_0, t - \tau (z_0, z)) - q \frac{\partial f_{\pm} (\mu, W)}{\partial W} \int_{z_0}^{z} dz' \delta E_z (z', t - \tau (z', z)) \]

(A6)

If we now assume that the wave fields and the perturbed distribution function oscillate at a frequency \( \omega \), we can cancel out the \( \exp(-i\omega t) \) time dependence of each term and write
\[ \delta f_\pm(z) = \delta f_\pm(z_0) e^{i\omega t(z,|z|)} - q \frac{\partial f_\pm}{\partial W} \int dz' \delta E_z(z') e^{i\omega t(z',z)} \]  

(A7)

Considering a bounded system between \( z = 0 \) and \( z = L \), we can write the two solutions as

\[ \delta f_+(z) = \delta f_+(0) e^{i\omega t(0,z)} - q \frac{\partial f_+}{\partial W} \int_0^L dz' \delta E_z(z') e^{i\omega t(z',z)} \]  

(A8)

\[ \delta f_-(z) = \delta f_-(L) e^{i\omega t(L,z)} + q \frac{\partial f_-}{\partial W} \int_z^L dz' \delta E_z(z') e^{i\omega t(z',z)} \]  

(A9)

Note the change of sign in equation (A9) due to the switch in the limits of integration. It can be seen that if we know the parallel electric field as a function of position, these integrals can be evaluated.

Next we must evaluate the field-aligned current. This can be found be integrating the perturbed distribution function over all velocities, which gives

\[ \delta j_z(z) = q \int_0^\infty dv_z v_z \delta f_+(x,v_z) + q \int_0^\infty dv_z v_z \delta f_-(x,v_z) \]  

(A10)

This can be written in terms of a single integral by defining \( v = v_z \) in the first term and \( v = -v_z \) in the second term

\[ \delta j_z(z) = q \int_0^\infty dv \left[ \delta f_+(z,v) - \delta f_-(z,-v) \right] \]  

(A11)

Since the distribution function can be represented as an integral over \( z' \) of the parallel electric field as in equations (A8) and (A9), the field-aligned current can be written in the form

\[ \delta j_z(z) = \int dz' \sigma(z,z') \delta E_z(z') \]  

(A12)

Note that in general terms relating to the boundary terms in equations (A8) and (A9) must be added to equation (A12). If the field-aligned current can be determined from the wave equations, then the integral equation (A12) can then be solved for the parallel electric field.

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Figure Captions

Figure 1. Normalized power dissipation to the kinetic dispersion relation for a uniform plasma with equal temperatures, as a function of $k_{ze}c/\omega_{pe}$ and $V^2_{ce}/V^2_A = T_e/m_eV^2_A$ for temperature ratios of (a) 1; (b) 2; (c) 5; (d) 10. Note that the power dissipation is favored by cooler ions.

Figure 2. Plot of the dimensionless integral in equation (33) that gives the conductivity kernel as a function of the parameter $\omega |z - z'|/a_e$. The solid curve gives the real part of the integral, while the dotted line gives the imaginary part. Mixing between particles of different velocity causes this kernel to go to zero when the distance between the points is greater than the distance a thermal particle travels in one wave period.

Figure 3. The model Alfvén speed profile as given by equation (20) for a value of $\epsilon = 0.1$.

Figure 4. The effective normalized conductivity as defined by equation (39) for a model with $\epsilon = 0.1$, $k^2\lambda^2_M = 0.01$ and $2\hbar\omega/V_{Al} = 2.4$ for various thermal speeds $a_e/V_{Al} = (a) 1.0; (b) 3.0; (c) 10.0; and (d) 30.0$. Solid line gives the real part while the dotted line gives the imaginary part. Note that negative imaginary parts in this presentation indicate regions of dissipation.

Figure 5. Relative dissipation normalized to the input Poynting flux for the ionospheric Alfvén resonator models shown in Figure 5. Here the solid line is for $a_e/V_{Al} = 1.0$, the dotted line for 3.0, the dashed line for 10.0 and the dash-dot line for 30.0. Note that in addition to the region of Landau damping in the uniform region on the right half of the figure, there is additional dissipation in the region of the Alfvén speed gradient for $z/2\hbar < 5$. 
Figure 1. Normalized power dissipation to the kinetic dispersion relation for a uniform plasma with equal temperatures, as a function of $k_x c / \omega_p$ and $v_e^2 / V_A^2 = T_e / m_e V_A^2$ for temperature ratios of (a) 1; (b) 2; (c) 5; (d) 10. Note that the power dissipation is favored by cooler ions.
Figure 2. Plot of the dimensionless integral in equation (33) that gives the conductivity kernel as a function of the parameter $ω|z - z'|/a_c$. The solid curve gives the real part of the integral, while the dotted line gives the imaginary part. Mixing between particles of different velocity causes this kernel to go to zero when the distance between the points is greater than the distance a thermal particle travels in one wave period.
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