Non-local interactions between electrons and Alfvén waves on auroral field lines

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Abstract

The interaction of auroral electrons with kinetic Alfvén waves is complicated by the fact that the Alfvén speed above the ionosphere is strongly inhomogeneous, leading to a region often referred to as the ionospheric Alfvén resonator (IAR). For these waves, the wave-particle interaction must be treated with a non-local kinetic approach. Linear damping of these waves due to the wave-particle interaction has been calculated based on a dipolar field line model; however, particle orbits in the wave field are not well described by perturbation theory due to the development of new turning points in the particle trajectories, especially for low-energy particles. Full orbit calculations of such particles have been performed in order to assess the validity of the linear theory and to determine how much wave energy is converted into electron energy flux that is precipitated into the ionosphere. In addition, the precipitating electrons show a phase shift with respect to the field-aligned current in the waves, leading to a modification of theories of ionospheric feedback in the ionospheric Alfvén resonator.
1. Introduction

Observations from the FAST satellite [Chaston et al., 1999, 2000, 2002a,b] have indicated that precipitating auroral electrons often are field-aligned and have a broad distribution in energy, in contrast to the typical auroral “inverted-V” type precipitation, which have a characteristic energy but are broad in pitch angle. Regions of this type of field-aligned acceleration have been observed throughout the auroral zone but are often seen at the polar cap boundary of the auroral zone. Similar observations of field-aligned electrons have been seen over many years, particularly from sounding rocket missions [Johnstone and Winningham, 1982; Arnoldy et al., 1985; Robinson et al., 1989; McFadden et al., 1986, 1987, 1990, 1998; Clemmons et al., 1994; Lynch et al., 1994, 1999; Knudsen et al., 1998]. These field-aligned distributions are not consistent with the usual picture of a plasma sheet distribution that has been accelerated in a quasi-static potential drop. Rather, test particle models invoking the electron inertial effect have suggested that low frequency waves are responsible for this acceleration; these waves may be either electromagnetic ion cyclotron waves [Temerin et al., 1986] or Alfvén waves [Kletzing, 1994; Thompson and Lysak, 1996; Chaston et al., 1999, 2000, 2002a,b; Kletzing and Hu, 2001].

Recently, non-local kinetic models have been developed to determine the wave-particle interactions more self-consistently, both in the context of field line resonances [Rankin et al., 1999; Tikhonchuk and Rankin, 2000, 2002] and for the ionospheric Alfvén resonator [Lysak and Song, 2003a,b]. This work has shown that kinetic effects can produce larger parallel electric fields than those associated with the electron inertial effect computed using two-fluid theory. The latter papers provided a self-consistent solution to the wave equations including the linearized electron kinetics along the field line, and calculated the relative amount of input wave energy that is reflected, absorbed by the wave-particle interaction, and dissipated as Joule
heating in the ionosphere.

However, this work is incomplete in a number of ways. First of all, as in all linearized Vlasov models, the perturbed distributions were calculated by an integral over unperturbed electron trajectories. However, for particles near their mirror point or those with low energies, a small perturbation in the parallel electric field can lead to either a new turning point or a large deviation of the turning point from its unperturbed value. In addition, although the linearized calculation can indicate that wave energy is being dissipated, the fate of this energy is not determined. A calculation of the amount of the wave energy that can be converted into precipitating electron energy flux, where it can cause auroras and modify the ionospheric conductivity, would be most useful.

It is the purpose of this paper to present such calculations, compare the results to the linear theory and discuss implications for the formation of the aurora and modification of the ionosphere. The theoretical foundation of the model will first be briefly discussed. Then, calculations of perturbed particle trajectories for various cases will be shown. The energy in precipitating particles will be determined and compared to the results of the linear theory. Finally, consequences of this model for the development of ionospheric feedback instabilities will be discussed.

2. Theoretical Development

The development of the model follows the results presented in Rankin et al. [1999], Tikhonchuk and Rankin [2000, 2002], and Lysak and Song [2003a,b]. The details of the model can be found in those works; here we will briefly sketch the theory behind these calculations.

The basic concept of this model is to consider the wave equations for Alfvén waves along
auroral field lines, including a fully kinetic treatment for the electron motion along the field line. The wave equations for the kinetic Alfvén wave can be expressed by assuming a wave of constant frequency and perpendicular wave number (which is assumed to be in the $x$ direction).

In this case, Faraday’s Law and the perpendicular and parallel components of Ampere’s Law can be written as

\[
\begin{align*}
\frac{\partial E_x}{\partial z} &= i\omega B_y + ik_{\perp} E_z & (1) \\
\frac{\partial B_y}{\partial z} &= \frac{i\omega}{c^2} \left( 1 + \frac{c^2}{V_{A}^2} \frac{1 - \Gamma_0 (\mu_i)}{\mu_i} \right) E_x & (2) \\
-i\omega \epsilon_0 E_z + \int dz' \sigma (z,z') E_z (z') &= \frac{ik_{\perp} B_0}{\mu_0 B_{0f}} B_y & (3)
\end{align*}
\]

It should be noted that in these equations, $E_x$, $B_y$ and $k_{\perp}$ are mapped to their ionospheric values using an isotropic dipolar mapping. Thus, the only mapping factor that appears explicitly is the ratio of the background magnetic field strength to that in the ionosphere, $B_0 / B_{0f}$, that appears in equation (3). The ion gyroradius correction is included in equation (2) through the Bessel function $\Gamma_0$ which is a function of the parameter $\mu_i = k_{\perp}^2 \rho_i^2$. It may be noted that this term is sometimes approximated by the relation $(1 - \Gamma_0 (\mu_i))/\mu_i \approx 1/(1 + \mu_i)$, as in Johnson and Cheng [1997]; however, in this work we retain the full Bessel function expression.

The electron kinetic effects enter into the wave equations above through the non-local conductivity relation that gives the field-aligned current in the parallel Ampere’s Law, equation (3). This current can be defined in the usual way by calculating the first velocity moment of the perturbed distribution function. Since the perturbation in the distribution function is linear in the parallel electric field, the field-aligned current can be written as a non-local conductivity relation.
\[ \delta j_z(z) = \int_0^L dz' \sigma(z, z') \delta E_z(z') \] (4)

The conductivity kernel \( \sigma(z, z') \) can be determined by solving the Vlasov equation for electrons in a dipolar magnetic field subject to the propagation of a kinetic Alfvén wave [Lysak and Song, 2003b]. Since these waves are at very low frequencies, the magnetic moment of the electrons will be conserved. In addition, the total energy of an electron is conserved in the absence of the wave; however, the parallel electric field of the wave will modify this total energy. The linearized Vlasov equation can be solved in the usual way by integrating along the unperturbed orbits defined by constant magnetic moment and energy.

Now we must consider the types of particle trajectories that are present. In general, there a number of particle populations on an auroral field line due to the interplay between the magnetic mirror force and the parallel electric field [e.g., Whipple, 1977; Chiu and Schulz, 1978]. However, for the initial results presented here, we will assume that there is no parallel electric field. In this case, there are only three types of electron trajectory, each with their own background distribution: (1) ionospheric electrons moving upward; (2) magnetospheric electrons that precipitate into the ionosphere; and (3) magnetospheric electrons that mirror at some altitude and return to the magnetosphere.

The wave mode structure of the Alfvén wave with the electron kinetic effects included can be found by solving equations (1)-(3) with the conductivity kernel found by solving the Vlasov equation. These equations are solved as follows. First, the equilibrium distributions are determined, which give the Alfvén speed profile and the non-local conductivity kernel. Then, equations (1) and (2) are solved by integrating upwards from the ionosphere, where the two fields are related by the ionospheric boundary condition, \( B_y + \mu_0 \Sigma_p E_x = 0 \). For this first
approximation, the parallel electric field is determined from the two-fluid model. The value of $B_y$ obtained in this calculation is then used to solve the integral equation (3) using the Nystrom method with Gaussian integration and zero-order regularization as described by *Delves and Mohamed* [1985]. The new value of $E_z$ obtained from this solution is then used to integrate equations (1) and (2) again. This procedure is repeated until convergence is reached. The solution is checked by monitoring the conservation of energy in the system: the input Poynting flux averaged over the wave cycle must equal the sum of the Poynting flux reflected out the upper boundary, the ionospheric Joule dissipation, and the parallel dissipation due to the wave-particle interaction, or symbolically

$$S_{inc} = S_{ref} + \frac{1}{2} \sum \left| E_{sl} \right|^2 + \frac{1}{2} \int dz \text{Re} \left( j_z E_z^* \right)$$  (5)

Plots showing the relative magnitudes of the three terms on the right-hand side of equation (5) have been presented by *Lysak and Song* [2003b].

### 3. Particle Trajectories

In order to assess the validity of the linear theory we have performed trajectory calculations based on the linear wave fields. The standard model that will be considered consists of a cold plasma population in the topside ionosphere that has an ionospheric density of $n_0 = 10^5$ cm$^{-3}$ and a scale height of 600 km that is assumed to be composed of oxygen ions. The “hot” plasma, by which in this context we mean the part of the distribution that will be treated kinetically, is assumed to have a density of 1.0 cm$^{-3}$ and an isotropic electron temperature of 500 eV, although this temperature will be varied in some of the cases shown below. The perpendicular ion temperature is assumed to be 3 keV for calculating the ion gyroradius. The ion
population that neutralizes these electrons is assumed to be composed of protons. These parameters are typical of plasma sheet parameters; although, of course, they may vary in different circumstances. These parameters will be used in all of the runs below unless otherwise stated.

The other important parameters that will be varied are the perpendicular wavelength and frequency of the wave, and the ionospheric Pedersen conductivity. In all cases shown in the present paper, we take the perpendicular wavelength mapped to the ionosphere to be 10 km. We consider two values for the Pedersen conductivity: 1.9 mho and 10 mho. The value of 1.9 mho is chosen since it matches the Alfvén admittance at the ionospheric end of the field line, while the other case illustrates the effects of higher conductivity. It should also be noted that the top end of the computational box is placed at 4 R_E geocentric altitude. This location is somewhat arbitrary; however, we shall see that most of the dissipation occurs below this altitude for the runs considered.

Detailed solutions for the wave structure in these cases was presented by Lysak and Song [2003b]. Figure 1 shows the parallel electric field profiles for 2 runs, one at 0.22 Hz and the other at 0.35 Hz. The conductivity is set at 1.9 mho in these runs. These frequencies were chosen since they correspond to a maximum and minimum, respectively, of the absorption due to the wave-particle interaction. The panels in this figure are reproduced from Figures 3b and 4b of Lysak and Song [2003b] and are included here for reference. In this figure, the solid and dashed lines give the real and imaginary part of the parallel electric field, while the dashed and dash-dot curves give the real and imaginary parallel electric field for the cold plasma approximation. It is useful to note that the total integrated parallel potential drop in these fields are 307 V and 125 V, respectively, although of course these are time-dependent fields and so the instantaneous
potential is only relevant for fast particles that pass through the system in a time much less than the wave period.

Figure 2 shows characteristic trajectories of field-aligned electrons starting at the ionosphere with 1, 10, 100 and 1000 eV, respectively, for the 0.22 Hz case. Since the wave field is time dependent, the electron trajectory depends on the time that the particle is injected into the system. Thus, 8 curves are given for each energy, corresponding to 8 equally spaced phases with respect to the wave. It can be seen that the particle trajectory is a sensitive function of this phase, especially for lower energies. The 1-eV electrons that are injected are all reflected by the wave field, and are dispersed in energy, with some electrons gaining an energy up to 50 eV in the wave particle interaction, although some electrons actually lose energy. At intermediate energies, 10 eV and 100 eV, some electrons are reflected while some pass through the wave and reach the upper boundary. It can be seen that some electrons are trapped for one or more wave periods as they pass through the wave. At the highest energy shown, 1 keV, the electrons all pass through the wave structure and gain or lose an energy of up to 300 eV, consistent with the total potential in the wave. These electrons pass through the region in about 1 second, compared to the 4.5 second period of this wave.

Figure 3 shows the trajectories of magnetospheric electrons for this case. Panels (a) and (b) show 100 eV electrons, while (c) and (d) give the trajectories for 1 keV electrons. Panels (a) and (c) are for field-aligned electrons, while the other two panels give a case where the magnetic moment expressed in eV/μT is one tenth of the total energy in eV. Thus, in the absence of the wave these particles would mirror where the magnetic field is 10 μT, which is at 1.79 R_E in this model. At the top boundary of the system at 4 R_E, these particles have a pitch angle of 17.5°. It can be seen from this figure that the field-aligned particles can be reflected by the waves at lower
energies, but not at the higher energy. Similarly, the higher-energy electrons with a finite pitch angle have their mirror points changed slightly by the wave, while lower-energy electrons are more strongly affected, with their mirror points being raised or lowered as their parallel energy is reduced or increased, respectively, by the wave field. Indeed, some of these particles almost reach the ionosphere in this case; particles with a slightly more field-aligned pitch angle or in a larger wave field will precipitate.

These results point out a major limitation of the linear kinetic model used here: the particle trajectories can vary greatly from the linear trajectories used to compute the wave fields. The presence of new turning points introduced by the wave field and the possibility that some particles can be trapped in the wave field combined with the mirror force are not taken into account in the linear calculations and would introduce significant difficulties in the calculation. It should be noted that these difficulties would increase as the wave amplitude is increased. The results presented here are for a modest case where the input Poynting flux is 1.45 mW/m². To make a more quantitative assessment of the usefulness of the linear theory, we will compare the predictions of the linear results with the results of the trajectory calculations in the next section.

4. Global energy balance: test particle model

The results presented in section 2 above as well as those in Lysak and Song [2003a,b] were based on a linearized kinetic model of the electron-Alfvén wave interaction. As was seen in the previous section, however, the actual trajectories of particles in the wave fields can deviate significantly from the linear trajectories. In particular, the presence of the wave parallel electric field can introduce new turning points in the trajectories, and particles may even be quasi-trapped for a number of cycles before exiting the system. Thus, it is worthwhile to investigate
how well the linear results can represent the more realistic situation.

In addition, the linear results presented above indicate how much energy is lost to the wave in the interaction, but not the fate of that energy. From the point of view of auroral physics, we are particularly interested in how much of this energy is precipitated into the ionosphere where it can ionize the neutral atoms as well as excite these atoms to produce the auroral light.

To answer these questions, we have performed a series of runs based on the models above. We consider a Maxwellian plasma sheet plasma with a density of 1 cm\(^{-3}\) and a temperature of 1 keV, and a perpendicular wavelength of 10 km. The conductivity is taken to be 1.9 mho or 10 mho, and the frequency is set at 0.225 Hz or 0.35 Hz, covering a wide range of dissipation parameters. The energy flux of precipitating particles is found by integrating the trajectories backward from the ionosphere to the source (which may be either in the ionosphere or at the top boundary) and weighting them by the distribution function at the source using Liouville’s theorem. Forty exponentially spaced values of each of energy and magnetic moment are used, and twenty phases of the wave are considered. It was verified that increasing these numbers does not significantly change the results. The wave fields were normalized to an incident Poynting flux of 10\(^{-4}\) mW/m\(^2\), a very low value that might be expected to be in the linear regime, as well as a stronger but still modest Poynting flux of 1 mW/m\(^2\), near the lower threshold of typical auroral fluxes.

Table 1 shows the results from these calculations. In this table the columns labeled “Ionosphere” give the relative energy flux dissipated in the ionosphere, while the columns labeled “Magnetosphere” give the energy flux reflected back to high altitudes. These are summed together in the “Total” column for the two values of the input Poynting flux. By
comparing these total values to the linear values, it can be seen that the linear results are approximately recovered by the direct orbit calculations for the lower Poynting flux value in three of the four cases, with the exception being the $\Sigma_p = 10$ mho, $\nu = 0.35$ Hz case. The discrepancy in this case is probably due to the fact that this case corresponds to a sharp minimum in the dissipated energy flux according to linear theory [cf. with Figure 2b of Lysak and Song, 2003b], and so a slight error in the frequency changes in predicted energy dissipation by a relatively large amount. It can also be seen that the total dissipation calculated from the particle trajectories depends on the amplitude of the wave. This is due to the deviation of the particle trajectories from the linear trajectories in this higher amplitude case.

The “Ionosphere” columns of Table 1 show the fraction of the input Poynting flux that is precipitated into the ionosphere. In all of the cases only a few percent of the incident Poynting flux is converted into precipitating electron energy flux that could potentially excite the aurora or ionize the neutral atmosphere. These results suggest an efficiency of 1-10% in producing electron precipitation from incident Poynting flux. The rest of the dissipated energy goes into particles that escape into the outer magnetosphere. Since these particles represent only a small distortion of the original Maxwellian distribution, this energy would be difficult to identify from particle observations.

5. Phase dependence of precipitation

The considerations of the previous sections have focused on the energy fluxes averaged over a wave period. However, due to the time-dependent nature of the wave fields, the amount of energy precipitated is not constant during the wave cycle. As can be seen in Figure 1, the wave parallel electric field is concentrated at higher altitude, and the electron transit time from
the acceleration region to the ionosphere is a few seconds, comparable to the wave periods that are being considered. It is useful to note that a 100 eV electron travels approximately 1 Re/s.

Figure 4 shows the precipitated energy flux as a function of wave phase for the (a) 0.225 Hz and (b) 0.35 Hz cases for a Pedersen conductivity of 1.9 mho and an input energy flux of 10 mW/m². Thus these cases correspond to the middle two rows of Table 1, but with the fields scaled up to give the higher input Poynting flux. The dashed line in these figures gives the energy flux in the absence of the wave, i.e., this is the precipitation in the unperturbed loss cone, which has a value of 1.69 mW/m² for these cases. The dot-dashed line gives the average over the wave period; thus, the difference between the dot-dashed and dashed lines gives the average enhancement due to the wave that is tabulated in Table 1. The solid curves give the energy flux at each phase of the wave cycle.

Two important features show up in these figures. First of all, while the average energy flux enhancement is only a few percent of the input Poynting flux (0.65 mW/m² and 0.30 mW/m² for the two cases), the peak precipitation is much larger, being 3.93 mW/m² and 2.13 mW/m² above the “no wave” case in the two figures. Thus, up to 40% of the wave Poynting flux is being converted to ionospheric precipitation during the peak phase of the wave. This suggests that the Alfvénic acceleration of electrons can be quite efficient during the peak phases. Such a situation may manifest itself as fluctuating aurora with periods of a few seconds.

These results also have implications for theories of the ionospheric feedback instability [e.g., Atkinson, 1970; Holzer and Sato, 1974; Sato, 1978; Miura and Sato, 1980; Lysak, 1991; Pokhotelov et al., 2002; Lysak and Song, 2002], in which conductivity fluctuations lead to secondary field-aligned currents which further modify the ionosphere. While the most basic form of this instability simply considers the inflow and outflow of electrons to modify the
conductivity, the instability can be greatly accelerated by invoking the precipitation of hot electrons to provide the additional ionization to drive the instability. Both linear stability analysis and nonlinear numerical simulations have usually assumed that this precipitation arrives in phase with the upward field-aligned current.

The results shown in Figure 4 call this assumption into question. The dotted line in the two panels of the figure give the normalized profile of the field-aligned current from the model, displaced upward by one unit to fit on the figure. This curve is greater than one in regions of upward field-aligned current, while the values are lower than one in the downward current region. The maximum upward field-aligned current is thus at a phase of $\pi/2$ in these figures; thus it would be at this point that the maximum ionization would take place in the standard feedback theory. It is clear that there is a frequency-dependent phase shift between the upward current maximum and the peak precipitation. Note that the hot precipitating electrons carry only a small amount of the field-aligned current in this case; most of the current is carried by the cold ionospheric electrons that have a much greater density at the ionosphere than the hot electrons.

An estimate of the possible role of this phase shift in feedback theories can be found following the formulation of Lysak and Song [2002]. The ionization rate due to hot particle precipitation can be written as $S = \varepsilon_\parallel / E_0 \Delta z$, where $\Delta z$ is the thickness of the ionosphere and $E_0 \approx 35$ eV is the energy required to create an electron-ion pair [Rees, 1963]. In the linear feedback theory, this source is usually parameterized by a parameter $\gamma$ and written as $S = \gamma j_\parallel / e \Delta z$, and so the energy flux is effectively replaced by $\gamma E_0 j_\parallel / e$. Thus according to this relation, $\varepsilon_\parallel$ and $j_\parallel$ are in phase with each other. A phase shift can be taken into account by allowing $\gamma$ to be a complex number, $\gamma = \gamma_0 e^{i\phi}$. Now, in the conventional feedback theory, the growth rate $\Gamma$ is given by the expression
\[ \Gamma = -\frac{\gamma k_{\perp} \cdot u_d Z^* \Sigma_{P0}}{|1 + Z \Sigma_{P0}|^2} - v_r \]  

(6)

Here \( \Sigma_{P0} \) is the background Pedersen conductance, \( u_d \) is the relative drift between ions and electrons in the ionosphere, \( v_r \) is the recombination rate and \( Z = Z' + iZ'' \) is the complex impedance of the flux tube, which depends on the Alfvén speed profile. This impedance is purely imaginary with a negative imaginary part for an ideal standing field line resonance or for waves in a strong ionospheric Alfvén resonator [Lysak and Song, 2002].

If the parameter \( \gamma \) is allowed to be imaginary, the growth rate given by equation (6) is modified to read

\[ \Gamma = \frac{\gamma_0 k_{\perp} \cdot u_d}{|1 + Z \Sigma_{P0}|^2} \left( -Z^* \Sigma_{P0} \cos \varphi + [1 + Z \Sigma_{P0}] \sin \varphi \right) - v_r \]  

(7)

Since the real impedance \( Z' \) is usually small, the second term in parentheses is usually of order unity while the first term \( Z^* \Sigma_{P0} \sim \Sigma_{P0} / \Sigma_{A0} \) is usually much greater than 1. Thus, the cosine factor will generally reduce the growth rate and could switch the sign of this term leading to stability. Physically, Figure 4 shows that the additional precipitation actually occurs in the downward current region, so that the additional ionization due to precipitation may be somewhat offset by the outflow of ionospheric electrons carrying this downward current. While there may be circumstances under which the instability persists or is even enhanced, the conclusion is that the phase shift of the precipitation will generally be stabilizing.

6. Conclusions and Future Work

Models of auroral particle acceleration have generally been associated with one of two approaches: a kinetic approach based on particle trajectories [e.g., Knight, 1973; Chiu and
Schulz, 1978; Fridman and Lemaire, 1980] or on a fluid approach emphasizing the evolution of fields and currents [e.g., Mallinckrodt and Carlson, 1978; Goertz and Boswell, 1979; Lysak and Dum, 1983]. Both approaches have their advantages and drawbacks. While the kinetic theories can describe the details of the electron distribution and more accurately describe the response of the electrons to parallel electric fields and the mirror force, they have generally been formulated in the steady-state and so cannot describe the evolution and development of the parallel electric fields. On the other hand, fluid models have been successful in following the evolution of the fields and currents, but do not describe the electron distribution exactly. This paper is the extension of a new hybrid approach [Rankin et al., 1999; Tikhonchuk and Rankin, 2000, 2002; Lysak and Song, 2003a,b] that attempts to include both time-dependent fluid effects as well as the kinetics of the auroral electrons.

This paper has presented a number of results based on following particle trajectories in fields based on a kinetic model of Alfvén waves along auroral field lines. While the linear theory can give an indication of kinetic effects in a linear model, such a theory is based on integration of the Vlasov equation over unperturbed trajectories. In a mirror geometry such as the auroral zone, even weak parallel electric fields can disrupt such unperturbed trajectories, most importantly by introducing new mirror points that can reflect particles, causing a large deviation from the trajectories assumed in the linear theory.

Despite this difficulty, the work presented here shows that the linear theory does give a reasonable description of the dissipation produced by the wave-particle interaction due to these Alfvén waves. For very weak fields, results from the trajectory calculations reproduce the linear results, and give additional information in terms of how much of the dissipated energy can be deposited in the auroral ionosphere. Results for stronger waves show that the linear theory still
gives a qualitative description of the amount of energy dissipation, in that cases with high linear dissipation also correspond to cases with high dissipation as shown by the trajectory calculations. These results indicate that on average, up to 10% of the Poynting flux incident in the form of Alfvén waves can be converted to ionospheric precipitation.

Furthermore, these calculations also indicate that this energy flux does not arrive at the ionosphere uniformly over the wave period, but rather in bursts occurring once per wave period. These bursts can instantaneously deposit as much of 40% of the incident wave Poynting flux into the ionosphere. Such bursts can produce transient aurora as well as enhance the ionospheric conductivity on time scales short compared with the recombination time. Thus, these bursts may lead to feedback interactions on auroral field lines.

However, the phase shift of these bursts with respect to the peak in upward field-aligned current calls into question theories and numerical models of the ionospheric feedback instability. The qualitative considerations presented above suggest that the phase shift may be a stabilizing influence; however, the theory of the feedback instability is still in rudimentary form (e.g., assuming a height-integrated conductivity model), so that the role of the feedback instability in auroral dynamics is still an interesting but unsolved question. Clearly, the present work indicates that a more complete theory of this instability will require a combination of a better ionospheric model plus a kinetic model for the propagation of accelerated electrons along auroral flux tubes.

The kinetic model presented here and in earlier papers represents only a first step toward such a complete model. This model is limited in that it is based on a sinusoidal wave perturbation in time and in the perpendicular direction, while real auroral Alfvén waves are likely be in the form of wave packets or wave fronts [Song and Lysak, 2001]. The present model is based on an equilibrium with no background parallel electric field; a model including
static parallel electric fields should be a straightforward extension that will be the topic of future work. Finally, the nonlinear dynamics of the particles and waves must be included, since the wave fields themselves can drastically alter the particle trajectories and thus the non-local conductivity kernel that is central to the kinetic theory. This extension will require a significant amount of future work; however, the result of such a model would be a non-local, non-linear, time-dependent kinetic model that would give a unified picture of auroral acceleration encompassing both fluid and kinetic aspects of this problem.

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References


Lynch, K. A., D. Pietrowski, R. B. Torbert, N. Ivchenko, G. Marklund, and F. Primdahl,
Multiple-point electron measurements in a nightside auroral arc: Auroral Turbulence II

Lysak, R. L., Feedback instability of the ionospheric resonant cavity, *J. Geophys. Res.*, 96, 1553,

Lysak, R. L., and C. T. Dum, Dynamics of magnetosphere-ionosphere coupling including


Lysak, R. L., and Y. Song, Kinetic theory of the Alfvén wave acceleration of auroral electrons, *J.

Lysak, R. L., and Y. Song, Nonlocal kinetic theory of Alfvén waves on dipolar field lines, *J.

Mallinckrodt, A. J., and C. W. Carlson, Relations between transverse electric fields and field-

McFadden, J. P., C. W. Carlson, and M. H. Boehm, Field-aligned electron precipitation at the

McFadden, J. P., C. W. Carlson, M. H. Boehm, and T. J. Hallinan, Field-aligned electron flux

McFadden, J. P., C. W. Carlson, and M. H. Boehm, Structure of an energetic narrow discrete arc,

Klumpar, E. G. Shelley, W. K. Peterson, E. Moebius, L. Kistler, R. Elphic, R.


Figure Captions

Figure 1. Parallel electric field profiles for runs with a frequency of (a) 0.22 Hz; (b) 0.35 Hz. Solid and dotted lines are real and imaginary part of the parallel electric field from the kinetic model, and dashed and dash-dot lines are the real and imaginary part from a cold two-fluid model.

Figure 2. Trajectories of ionospheric electrons injected into the auroral acceleration region with initial energies of (a) 1 eV; (b) 10 eV; (c) 100 eV; and (d) 1 keV. The different curves in each panel represent electrons emitted at various phases of the wave.

Figure 3. Trajectories of ionospheric electrons injected into the auroral acceleration region with initial energies and magnetic moments of (a) $W = 100$ eV, $\mu = 0$; (b) $W = 100$ eV, $\mu = 10$ eV/$\mu$T; (c) $W = 1$ keV, $\mu = 0$; and (d) $W = 1$ keV, $\mu = 100$ eV/$\mu$T. The different curves in each panel represent electrons emitted at various phases of the wave. Note that the electrons with $W/\mu = 10$ $\mu$T will mirror at a magnetic field of 10 $\mu$T, or an altitude of 1.79 R$_E$ in the absence of the wave.

Figure 4. Precipitation as a function of wave phase for the case with frequency (a) 0.225 Hz; (b) 0.35 Hz. Solid curve gives the precipitated energy flux. The dotted curve gives the normalized field-aligned current, displaced upward by one unit, with values greater than 1 indicating upward current. The dashed and dash-dot curve give the precipitated energy flux in the absence of the wave and the value averaged over the wave cycle, respectively.
Table 1. Comparison of linear model with trajectory calculations. Dissipation is given as a fraction of the incident Poynting flux

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<tr>
<td>10</td>
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<td>0.9773</td>
<td>0.8945</td>
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Figure 1. Parallel electric field profiles for runs with a frequency of (a) 0.22 Hz; (b) 0.35 Hz. Solid and dotted lines are real and imaginary part of the parallel electric field from the kinetic model, and dashed and dash-dot lines are the real and imaginary part from a cold two-fluid model.
Figure 2. Trajectories of ionospheric electrons injected into the auroral acceleration region with initial energies of (a) 1 eV; (b) 10 eV; (c) 100 eV; and (d) 1 keV. The different curves in each panel represent electrons emitted at various phases of the wave.
Figure 3. Trajectories of ionospheric electrons injected into the auroral acceleration region with initial energies and magnetic moments of (a) $W = 100$ eV, $\mu = 0$; (b) $W = 100$ eV, $\mu = 10$ eV/$\mu$T; (c) $W = 1$ keV, $\mu = 0$; and (d) $W = 1$ keV, $\mu = 100$ eV/$\mu$T. The different curves in each panel represent electrons emitted at various phases of the wave. Note that the electrons with $W/\mu = 10$ $\mu$T will mirror at a magnetic field of 10 $\mu$T, or an altitude of 1.79 $R_E$ in the absence of the wave.
Figure 4. Precipitation as a function of wave phase for the case with frequency (a) 0.225 Hz; (b) 0.35 Hz. Solid curve gives the precipitated energy flux. The dotted curve gives the normalized field-aligned current, displaced upward by one unit, with values greater than 1 indicating upward current. The dashed and dash-dot curve give the precipitated energy flux in the absence of the wave and the value averaged over the wave cycle, respectively.