Energetics of the ionospheric feedback interaction

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[1] The ionospheric feedback instability has been invoked as a possible mechanism for the formation of narrow auroral arcs. This instability can excite eigenmodes of both field line resonances and the ionospheric Alfvén resonator, producing narrow-scale structures. Although the basic dispersion relation of this instability has been discussed for both of these cases, the energetics of this instability has not been discussed quantitatively and questions remain as to the nonlinear evolution of this instability. The free energy for this instability comes from the reduction of Joule heating due to the preexisting convection caused by the self-consistent changes in ionization and conductivity due to Alfvénic perturbations on the ionosphere. In an active ionosphere, narrow-scale Alfvén waves can be overreflected; i.e., the reflected wave can have a larger amplitude than the incident wave, with the extra energy coming from a local reduction of Joule heating. Recombination produces a damping of this instability, particularly for high background conductivity, indicating that this instability operates best in a dark background ionosphere. This feedback interaction produces narrow-scale currents when strong gradients in the conductivity are produced, and effects from parallel resistivity or possibly kinetic effects will become important in its evolution. Theoretical constraints on low-spatial resolution observations of the energy dissipated by precipitation as opposed to Joule heating will be discussed. INDEX TERMS: 2407 Ionosphere: Auroral ionosphere (2704); 2409 Ionosphere: Current systems (2708); 2411 Ionosphere: Electric fields (2712); 2431 Ionosphere: Ionosphere/magnetosphere interactions (2736); KEYWORDS: ionospheric feedback instability, magnetosphere/ionosphere coupling, ionospheric Alfvén resonator, Joule heating, auroral arcs, field-aligned currents

1. Introduction

[2] The ionospheric feedback instability [Atkinson, 1970; Holzer and Sato, 1973; Sato, 1978] is a mechanism for the establishment of narrow-scale auroral arcs. This instability results from the change in the ionospheric conductivity that is produced by the self-consistent precipitation of electrons associated with the upward field-aligned current. In the presence of a large-scale convection electric field, this conductivity enhancement can produce secondary field-aligned currents that can in turn modify the ionospheric conductivity further. These secondary currents are associated with the emission of an upward propagating Alfvén wave; when this Alfvén wave is reflected back toward the ionosphere, either at the conjugate ionosphere or at large gradients in the Alfvén speed, a positive feedback can result if the phase of the reflected wave is such that the precipitating electrons associated with this wave impact the ionosphere at the previously established conductivity enhancement, increasing the conductivity further [Miura and Sato, 1980].

[3] Recent observations that discrete auroral arcs are suppressed when the underlying ionosphere is in daylight [Newell et al., 1996a, 1996b; Liou et al., 1997, 2001] have provided observational evidence that the ionospheric feedback mechanism may be an important mechanism for the formation of auroral arcs. Studies including seasonal and solar cycle variations [Cattell et al., 1991; Newell et al., 1998; Liou et al., 2001] have confirmed this general result, concluding that more auroral acceleration occurs in the winter and at solar minimum. Two possible explanations for these results are either that the lower conductivity under dark conditions enhances aurora, or alternatively, that the plasma density in the auroral acceleration region is lower under such conditions. Although it is certainly true that low densities in the acceleration region will enhance parallel electric fields [e.g., Song and Lysak, 2001], this paper will examine the first situation, regarding the effect of low conductivity on ionospheric feedback.

[4] The initial models of the feedback instability assumed that the Alfvén wave reflected from the conjugate ionosphere. It has been noted [Lysak, 1986, 1991] that the feedback instability can be greatly enhanced if reflections from the sharp gradient of the Alfvén speed above the auroral ionosphere are considered. The eigenmodes of this so-called ionospheric Alfvén resonator [Polyakov and Rapoport, 1981; Trakhtengertz and Feldstein, 1987, 1991] have periods of a few seconds, as opposed to the field line resonances invoked in previous work that have periods of many minutes. Thus the feedback instability operating in this resonator will evolve much more quickly than the instability based on field line resonators, and so the former may be called the fast feedback instability while the field line resonant version is a slow feedback instability. Com-
pressional effects on this instability have been included by Pokhotelov et al. [2000], who concluded that at the small spatial scales of this instability, such effects were probably not too important. D. Pokhotelov et al. (Harmonic structure of field line eigenmodes generated by ionospheric feedback instability, submitted to Geophysical Research Letters, 2001) (hereinafter referred to as Pokhotelov et al., submitted manuscript, 2001) have recently performed numerical simulations of this instability including a realistic Alfvén speed profile and found that both fast and slow feedback instabilities developed. They included non-symmetric ionospheres and verified that the feedback instability favored the ionosphere with lower background conductivity.

[5] These observational and theoretical studies point to the importance of such instabilities in the formation of auroral arcs but also raise more questions about the nature of these instabilities. A frequently asked question is the free energy source for the instability. While it has been stated that the energy comes from a reduced Joule dissipation [e.g., Lysak, 1991], a quantitative analysis of the energetics of the instability has not been published. The nonlinear evolution of such an instability has not been carefully considered. Finally, the consequences for this instability for observations of auroral precipitation and Joule heating should be clarified. The purpose of this report is to address these questions, resulting in a more complete physical picture of this instability and its consequences for magnetosphere-ionosphere coupling.

2. Basic Theory of the Feedback Instability

[6] The theory of the feedback instability has been presented in many of the above mentioned papers, but it is useful to summarize the basic results of this theory. The calculation will be done in the neutral rest frame, which due to the slow timescales for the evolution of the neutrals, will be assumed to be fixed during the calculation. It may be noted in this regard that Song et al. [2001] have suggested that it is more useful to formulate the Ohm’s law in the ionosphere in the plasma rest frame; however, in a time-dependent situation such as the one here, the plasma rest frame is continually changing. Thus we will adopt the traditional formulism based on the neutral frame, as discussed recently by Parker [1996] and Strangeway and Raeder [2001] and which has been the basis of numerical investigations of time-dependent processes in the ionosphere [Lysak, 1997, 1999]. In this frame, setting the z direction to be upward and the background magnetic field to be vertical, the current continuity equation together with the ionospheric Ohm’s law can be written as

\[
\frac{\partial}{\partial t} + \mathbf{u}_E \cdot \nabla \mathbf{j}_z = -\nabla \cdot (\Sigma_p \mathbf{E}_z - \Sigma_H \mathbf{E}_z \times \mathbf{b}) = -\Sigma_p \nabla \cdot \mathbf{E}_z - \mathbf{E}_z \cdot \nabla \Sigma_p - \mathbf{b} \times \nabla \Sigma_H.
\]  

(1)

Here \( \mathbf{b} \) is the unit vector in the direction of the magnetic field, which is \( \mathbf{b} = \pm \mathbf{z} \); in the Southern (Northern) Hemisphere, respectively. The second equation is the plasma continuity equation, which, if the ion and electron continuity equations are combined, becomes [Sato, 1978]

\[
\left( \frac{\partial}{\partial t} + \mathbf{u}_E \cdot \nabla \right) n = S - R \left( n^2 - n_0^2 \right),
\]  

(2)

where \( \mathbf{u}_E \) is the \( \mathbf{E} \times \mathbf{B} \) drift velocity, \( R \) is the recombination coefficient, \( n_0 \) is the background ionospheric density in the absence of precipitation, and \( S \) is the source term for the ionospheric density. If cold electrons carry the field-aligned current, then this source term simply represents the addition of electrons in upward current regions and the outflow of electrons for downward currents. Thus, in this case, \( S = j_x/e \Delta z \), where \( \Delta z \) is the ionospheric height. In addition, if there is energetic electron precipitation associated with an upward field-aligned current, there will be additional ionization caused by these energetic electrons. It has been estimated [Rees, 1963] that an additional electron-ion pair is produced in the ionosphere for every 35 eV of precipitated electron energy. In this case the source term can be written as \( S = j_x/e \Delta z + \epsilon_{\|}/\Delta z E_0 \), where \( \epsilon_{\|} \) is the precipitated energy flux and \( E_0 \approx 35 \text{ eV} \). For monoenergetic precipitating electrons that have fallen through a potential drop \( \Phi_{\|} \), the energy flux can be written as \( \epsilon_{\|} = \Phi_{\|} j_x \). To perform a linear analysis of the instability, it is convenient to write the source term as \( S = \gamma j_x/e \Delta z \), where \( \gamma = 1 + e \Phi_{\|}/E_0 \) is a factor that gives the number of electron-ion pairs produced per incident electron. Note that \( \gamma = 1 \) reduces to the cold electron case discussed above. Rees [1963] has noted that even for non-monoenergetic electrons, \( \gamma \) is roughly proportional to the average energy of the incident electrons. Note that the height-integrated conductances in equation (1) are related to the density in equation (2) through the relations \( \Sigma_{pH} = n e \Delta \zeta_{pH} \) where \( \mu_p \) and \( \mu_e \) are the Pedersen and Hall mobilities, which just depend on the neutral density and the plasma temperature and so are assumed to be constant. In this case, the continuity equation (2) can be written in terms of the Pedersen (or Hall) conductivities as

\[
\left( \frac{\partial}{\partial t} + \mathbf{u}_E \cdot \nabla \right) \Sigma_p = S_p - R_p (\Sigma_p^2 - \Sigma_n^2).
\]  

(3)

where the recombination coefficient \( R_p = R/e \mu_p \Delta z \) and the source term for the Pedersen conductivity can be written as

\[
S_p = \mu_p j_x + \frac{e \mu_p}{E_0} \| \mathbf{b} \|
\]  

(4)

[7] The initial equilibrium is assumed to be a uniform ionosphere, with conductances \( \Sigma_n \) and \( \Sigma_{p0} \), with a uniform convection electric field \( E_0 \) imposed. Linearizing equation (1) about this equilibrium gives the relation

\[
\delta j_x = -\Sigma_{p0} \mathbf{k}_\perp \cdot \delta \mathbf{E}_z - e \Delta z E_z \cdot \left( i \mathbf{k}_\perp \mu_p - \mu_e \mathbf{b} \times i \mathbf{k}_\perp \right) \delta n
\]  

(5)

where \( \mathbf{u}_d = \mu_p \mathbf{E}_z - \mu_e \mathbf{E}_z \times \mathbf{b} \) is the relative drift velocity between ions and electrons in the ionosphere. Linearizing equation (2) in the same way gives

\[
(\omega - \mathbf{k}_\perp \cdot \mathbf{u}_E + 2 i R n_0) \delta n = \left( \frac{J_p'}{e \Delta z} \right) \delta j_x
\]  

(6)

It will convenient to introduce the symbol \( \omega' = \omega - \mathbf{k}_\perp \cdot \mathbf{u}_E + i v_c \), where \( v_c = 2 R n_0 \) is the recombination rate. Note that since recombination is proportional to the square of the
density, this recombination rate is a function of the background density, or equivalently the background conductivity. Thus recombination is more rapid when the background conductivity is already high. Eliminating $\delta n$ from these relationships gives

$$
(1 - \frac{\gamma k_{\perp} \cdot u_a}{\omega}) \delta j_z = -i \Sigma_{p_0} k_{\perp} \cdot \delta E_{\perp}.
$$

(7)

It can be seen that in the absence of the zero-order current represented by $u_a$, equation (7) is simply the Fourier transform of the current continuity equation (1) for constant conductivities. Thus this additional term gives the effect of the changing conductivity in response to the field-aligned current.

[8] In order to complete a discussion of the feedback instability, we need to establish a second relation between the ionospheric resonator model and electric field. Following Sato [1978], we can define a magnetospheric impedance $Z$ by the relationship

$$
Z \delta j_z = i k_{\perp} \cdot \delta E_{\perp}.
$$

(8)

This relationship can be related to the dynamics of the shear Alfvén waves. For a purely upward (downward) propagating Alfvén wave, this impedance is given by $Z = \pm 1/k_{\perp}$, where $\Sigma_A$ is the Alfvén conductance defined by $\Sigma_A = 1/(\mu_0 V_A \sqrt{1 + \kappa_e^2 \lambda^2})$, where the electron inertial length $\lambda = c/\omega_{pe}$ has been included. If the Alfvén wave is reflected either from the conjugate ionosphere (in the case of a field line resonance) or from the Alfvén speed peak (in the case of the ionospheric resonator) with an electric field reflection coefficient of $R_c$, then the impedance becomes

$$
Z = \frac{1}{\Sigma_A} \frac{1 + R_c e^{i \omega T}}{1 - R_c e^{i \omega T}}.
$$

(9)

where $T$ is the travel time to the reflection point. This result reduces to the results of Sato [1978] for the case $R_c = 1$ that was assumed in that work, and to the result of Lysak [1986] for the generator conductivity model where $R_c = (\Sigma_A - \Sigma_G)/(\Sigma_A + \Sigma_G)$. However, it should be noted that equation (9) is more general, allowing for an arbitrary complex reflection coefficient such as the tau-generator model of Vogt et al. [1999]. It should also be noted that the ionospheric resonator model of Lysak [1991] can be included in this formalism. Using equation (11) of that paper and comparing with the form given in equation (8), we can write

$$
Z = \frac{i \Phi(x_0)}{\Sigma_A \Phi(x_0)} \approx \frac{J_0(2h_0 \omega/V_{A0})}{\Sigma_A J_1(2h_0 \omega/V_{A0})},
$$

(10)

where $\Phi(x_0)$, where $x_0 = 2h_0 \omega/V_{A0}$ is the eigenfunction for the potential at the ionosphere, which is given by $\Phi \propto J_{\text{int}}(x_0)$. Here $h$ is the density scale height above the ionosphere and $V_A$ is the Alfvén speed just above the ionosphere. The last expression in equation (10) holds in the limit of a very large Alfvén speed peak, $c = V_A/V_{A0} \ll 1$, where $V_A$ and $V_{A0}$ are the Alfvén speeds at the ionosphere and at the Alfvén speed peak, respectively.

[9] The dispersion relation for the feedback instability can be found by combining equations (7) and (8) and solving for the frequency, which gives

$$
\omega = k_{\perp} \cdot u_E - i \nu_r + \frac{\gamma k_{\perp} \cdot u_a}{1 + Z \Sigma_{p_0}}.
$$

(11)

The last term can give rise to an instability since the impedance $Z$ is a complex number. If we separate $Z$ into its real and imaginary parts, $Z = Z' + iZ''$, the growth rate $\Gamma$ of this instability becomes

$$
\Gamma = -\frac{\gamma k_{\perp} \cdot u_a Z' \Sigma_{p_0}}{|1 + Z \Sigma_{p_0}|} - \nu_r.
$$

(12)

Thus we can see that for waves propagating roughly in the direction of the drift, i.e., $k_{\perp} \cdot u_a > 0$, instability requires $Z'' < 0$. This corresponds to the reflected wave returning to the ionosphere with a phase shift corresponding to an inductive response, as noted by Miura and Sato [1980]. It should be noted that in the ionospheric resonator model, this instability condition implies that the $J_0$ and $J_1$ Bessel functions should have opposite sign; the first band in which this occurs is for $2.4 < 2h_0 \omega/V_A < 3.8$.

3. Physical Mechanism and Energetics

[10] Consideration of a number of simple cases will illustrate the energetics of this instability. First, it is instructive to consider the response of the ionosphere to an imposed density change. In the presence of the preexisting ionospheric current, the response to a density enhancement will be a combination of the reduction of the background electric field and the enhancement of the ionospheric current. To determine which of these responses is most important, we will consider the ionospheric continuity equation (1) without explicitly considering the density dynamics given by equation (2). To close the system, we will assume that an upward propagating Alfvén wave is emitted from the density perturbation, so that $\delta j_z = \Sigma_A \nabla \cdot \delta E_{\perp}$. Then, equation (1) can be written as

$$
(\Sigma_A + \Sigma_{p_0}) \nabla \cdot \delta E_{\perp} = -I_{\perp,0} \cdot \frac{1}{n_0} \nabla \delta n.
$$

(13)

where $I_{\perp,0}$ is the zero-order ionospheric current. If we assume that the perturbations only vary in the $x$ direction and that the perturbation is localized (or alternatively, periodic boundary conditions can be assumed), then the perturbation electric field can be found to be

$$
\delta E_x = -\frac{I_{\perp,0}}{\Sigma_A + \Sigma_{p_0}} \frac{\delta n}{n_0}
$$

(14)

and the field-aligned current becomes

$$
\delta j_z = -\frac{\Sigma_A}{\Sigma_A + \Sigma_{p_0}} I_{\perp,0} \cdot \nabla \frac{\delta n}{n_0}.
$$

(15)
Note that in the special case where the perturbation is in the direction of the Pedersen current, equation (14) can be written as

\[
\frac{\varepsilon E_0}{E_0} = \frac{\rho_0}{\rho_d + \rho_{00}} \frac{\delta n}{n_0}.
\]  

(16)

Equations (15) and (16) show that in the limit that \(\rho_d / \rho_{00} \ll 1\), the field-aligned current perturbation is small and the electric field is reduced. Thus the feedback instability is expected to be weak in this limit.

[11] Using the current continuity equation, the perturbation in the ionospheric current is given by

\[
\frac{\partial I_d}{\partial t} = \frac{\rho_d}{\rho_d + \rho_{00}} \frac{\delta n}{n_0}.
\]  

(17)

Using equations (14) and (17), we can calculate the total Joule heating including the perturbation

\[
I_{\perp} \cdot E_{\perp} = I_{\perp,0} \cdot E_{\perp,0} + I_{\perp,0} \cdot \delta E_{\perp} + \delta I_{\perp} \cdot E_{\perp,0} + \delta I_{\perp} \cdot \delta E_{\perp} = I_{\perp,0} \cdot E_{\perp,0} + \frac{\rho_d}{\rho_d + \rho_{00}} \frac{\delta n}{n_0} - \frac{\rho_d}{\rho_d + \rho_{00}} \frac{\delta n}{n_0} \frac{(\delta n)^2}{n_0^2}.
\]  

(18)

Note that when the Alfvén conductivity is smaller than the ionospheric conductivities so that \(\rho_d E_{\perp,0} < I_{\perp,0}\), both additional terms are negative for a positive density perturbation, indicating that the Joule heating is reduced by the conductivity enhancement. Averaging over a sinusoidal wave so that the linear term averages to zero, we find that the reduction in Joule heating is given by

\[
\Delta (I_{\perp} \cdot E_{\perp}) = -\frac{1}{2} \frac{\rho_d}{\rho_d + \rho_{00}} \frac{\delta n}{n_0} + \frac{1}{2} \frac{\rho_d}{\rho_d + \rho_{00}} \frac{\delta n}{n_0} \frac{(\delta n)^2}{n_0^2}.
\]  

(19)

where \(\delta n\) is the amplitude of the density perturbation. Thus a sinusoidal density fluctuation will lead to a decrease in the Joule heating. Note that this effect produces the strongest decrease in Joule heating when the background Pedersen conductance is equal to the Alfvén conductance.

[12] The energetics of the ionospheric feedback interaction can also be illustrated by considering the reflection of an incident Alfvén wave. The growing instability predicted by equation (12) would require that Alfvén waves that interact with the ionosphere should grow in amplitude. Such a condition is referred to as overreflection, and is often used to illustrate the mechanism of an instability. For example, Mann et al. [1999] have used such a formulation in a discussion of the Kelvin-Helmholtz instability. If we define a reflection coefficient by \(R = \varepsilon E_{\perp}^{up} / \varepsilon E_{\perp}^{down}\), and note that \(\delta E_{\perp}^{up} = \rho_d \nabla \cdot E_{\perp}^{up}\) and \(\delta E_{\perp}^{down} = -\rho_d \nabla \cdot E_{\perp}^{down}\), then superimposing the upgoing and downgoing waves, equation (7) can be written as

\[
\left[\rho_d \left(1 - \frac{\gamma k_{d,0} \cdot u_d}{\omega} \right) (R - 1) + \rho_{00} (R + 1) \right] \nabla \cdot \delta E_{\perp} = 0.
\]  

(20)

Solving this equation for the reflection coefficient gives

\[
R = \frac{\rho_d \left(1 - \frac{\gamma k_{d,0} \cdot u_d}{\omega} \right) (R - 1) + \rho_{00} (R + 1)}{\rho_d \left(1 - \frac{\gamma k_{d,0} \cdot u_d}{\omega} \right) (R + 1) + \rho_{00} (R - 1)}.
\]  

(21)

We may first of all note that in the absence of the preexisting convection, equation (22) is simply the standard reflection coefficient for Alfvén waves [e.g., Scholer, 1970; Malik and Carlson, 1978]. However, note that when \(\omega < \frac{\gamma k_{d,0} \cdot u_d}{\omega} \), the first terms in the numerator and denominator become negative. When this occurs, the magnitude of \(R\) can be greater than 1, and the wave can be said to be overreflected [e.g., Mann et al., 1999]. Noting that \(\omega\) is in general complex due to recombination, we can write

\[
\Sigma' + i \Sigma'' = \rho_d \left(1 - \frac{\gamma k_{d,0} \cdot u_d}{\omega} \right)
\]  

\[
= \rho_d \left(1 - \frac{\gamma k_{d,0} \cdot u_d}{(\omega - k_{d,0} \cdot u_d)^2 + \nu^2} \right) \left(\omega - k_{d,0} \cdot u_d - i \nu \right)
\]  

(23)

and then the magnitude of the reflection coefficient can be written as

\[
|R|^2 = \frac{(\Sigma' - \rho_{00})^2 + \Sigma''^2}{(\Sigma' + \rho_{00})^2 + \Sigma''^2} = 1 - \frac{4 \Sigma' \rho_{00}}{(\Sigma' + \rho_{00})^2 + \Sigma''^2}.
\]  

(24)

It can be seen that if \(\Sigma' < 0\), the magnitude of the reflection coefficient is greater than unity and the wave is overreflected. Note that if recombination can be neglected, this gives a condition for overreflection

\[
\omega < k_{d,0} \cdot (u_d + \gamma u_d).
\]  

(25)

Thus, for a given frequency there is a maximum wavelength for which the wave can be overreflected, indicating that the instability favors short wavelength waves. Note also that if the recombination damping is large compared to the wave frequency, the overreflection condition becomes difficult to achieve. In the context of the ionospheric resonator model, as we saw earlier instability requires \(2h_0 \omega / V_{d'} > 2.4\). In combination with equation (25), this implies that a necessary condition for instability is

\[
\frac{\gamma k_{d,0} \cdot u_d}{V_{d'} / 2h_0} > 2.4.
\]  

(26)

Numerical solutions of the growth rate for the feedback instability [Lysak, 1991] confirm that this condition is necessary for instability.

[13] At first glance, this overreflection may seem unphysical; however, the key to understanding this phenomenon lies in the reduction of Joule heating in an active ionosphere discussed above. We may note that the Poynting fluxes of the incident and reflected waves can be written as

\[
S_{\text{down}} = -\frac{1}{2} \frac{\partial E_{\perp}^{down}}{\partial t} \frac{\partial E_{\perp}^{down}}{\partial t}, \quad S_{\text{up}} = \frac{1}{2} |\rho_d E_{\perp}^{up}|^2.
\]  

(27)
Using equation (20) and current continuity equation, we can write
\[ \mathbf{I}_\perp = (1 - R)\Sigma_d \mathbf{E}_\perp^{\text{down}} \quad \mathbf{E}_\perp = (1 + R)\Sigma_d \mathbf{E}_\perp^{\text{down}}. \] (28)

Thus the change in the Joule heating is given by
\[ \Delta(\mathbf{I}_\perp \cdot \mathbf{E}_\perp) = \frac{1}{2} \text{Re}(\mathbf{I}_\perp \cdot \mathbf{E}_\perp^{*}) = \frac{1}{2} \left( 1 - |R|^2 \right) \Sigma_d |\mathbf{E}_\perp^{\text{down}}|^2. \] (29)

Equation (29) indicates that the Joule heating is reduced by the perturbation when the magnitude of the reflection coefficient becomes greater than unity. This reduction of the Joule dissipation accounts exactly for the amplification of the reflected wave, as can be seen from equations (27) and (29). It should be remembered that in the assumed equilibrium, there is a Poynting flux associated with the convection electric field that supplies the energy for the zero-order Joule heating. When the Joule heating is reduced below this zero-order value, this Poynting flux is released, resulting in the amplification of the reflected wave.

These considerations indicate that wave overreflection is a necessary condition for the feedback instability to operate. The presence of a minimum wave number for the instability as given by equation (25) was noted by Lysak [1991] in direct calculations of the growth rate in the ionospheric resonator model. In addition to the overreflection, the instability requires that the wave reflected from the magnetosphere have the proper phase relation as discussed above. Note, however, that even if the instability condition is not met, the overreflection of Alfvén waves that satisfy equation (25) indicates that such structures can be amplified and short wavelength fluctuations will gain energy due to the active ionosphere.

4. Nonlinear Evolution of the Feedback Instability

Satisfying the linear instability conditions discussed above is clearly necessary for the excitation of the feedback instability; however, this instability quickly enters a nonlinear stage, and in this case it is useful to turn to a numerical simulation. This simulation models equations (1) and (3) coupled to an MHD simulation of Alfvén wave propagation that has frequently been used to model magnetosphere-ionosphere coupling [Lysak and Dum, 1983; Lysak, 1985, 1986, 1997, 1999; Lysak and Hudson, 1987; Lysak and Song, 2000, 2001]. The model equations are given by equations (A2)–(A5) of the Appendix. The model is two-dimensional, and so the feedback instability is totally due to the Pedersen current rather than the Hall current. This simulation considers dipole field lines and a density profile that consists of an exponential profile of oxygen ions together with a power law profile of hydrogen, as given in Figure 1 of Lysak and Song [2000]. This model uses the displacement current in Ampère’s law to evolve the parallel electric field and the electron parallel equation of motion, including collisions, to evolve the field-aligned current as is described further in Lysak and Song [2001]. Details of this model are given in the Appendix. The electron collisions are given by an exponential profile with a scale height of 200 km and a value at the ionosphere of $10^4$ s$^{-1}$. An anomalous collision frequency depending on the local value of the electron drift velocity can also be included. The density profile used in the runs below profile gives an Alfvén speed at the ionosphere of 420 km/s and a scale height of 160 km, which gives a base frequency for the ionospheric resonator [Lysak, 1991] of $f_{av}/2\pi = 1.3$ s$^{-1}$. The Pedersen mobility is assumed to take on its peak value of $\mu_p = 10^5$ m$^2$/Vs, corresponding to the case $v_{in} = \Omega_i$ for a 0.5 G background magnetic field.

We first consider a run in which we model only the precipitation source due to the current itself, i.e., $\gamma = 1$, in order to observe the instability in its initial stages. In this case, the threshold for a wave with a wave number of 1 km$^{-1}$ as given by equation (26) is 350 mV/m. The initial Pedersen conductivity was 1 mho. This simulation is initiated with a uniform electric field of 500 mV/m (mapped to the ionosphere) through the whole box, with a pulse of amplitude 50 mV/m and width of 1 km turned on for 1 s at the top boundary to initiate the instability. Periodic boundary conditions in the horizontal direction are used. The spatial resolution in the horizontal direction is 25 m (2048 grid cells across the simulation). The ionospheric electric field, Pedersen conductivity, and total Joule heating are plotted in Figure 1 for three times early in the run, at $t = 4$, 6, and 8 s. Here the initial stages of the feedback instability can be seen. The initial field and current perturbation produces a wave that starts in the middle of the box and propagates to the right. It can be seen that the electric field and Pedersen conductivity vary roughly out of phase, while the field-aligned current is located at the gradients in the conductivity. This implies that the Pedersen current is enhanced in regions of larger conductivity. As the instability develops, the Pedersen conductivity develops steeper gradients, and the resulting field-aligned current becomes more spiky, as can be seen in Figure 2. Figure 3 shows the change in the total Joule heating for this run. It can be seen that the initial pulse enhances the Joule heating, but as the feedback instability develops, the Joule heating decreases, releasing its energy into the growing wave. Note that a value of 50 W/m represents an average of 1 mW/m$^2$ over the simulated ionosphere. It should be noted that a similar run with $\Sigma_P = 10$ mho showed no instability, with the initial perturbation decaying in time, while for $\Sigma_P = 0.1$ mho, the instability become very strong and numerical instabilities resulted.

This first run was set up to capture the evolution of the feedback instability in its linear stages. However, there are a number of unrealistic features of this run, such as the very large value of the convection electric field (500 mV/m). Typically, the convection electric field takes on more modest values (∼10 mV/m), and large electric fields (>100 mV/m) are spiky in nature with scale sizes the order of 10 km, as has been shown on a number of satellites [e.g., Mozer et al., 1980; Lindqvist and Marklund, 1990; Marklund et al., 1994; McFadden et al., 1999; Mozer and Hull, 2001]. Results from a run attempting to model such conditions are presented in Figure 4. This run shows the electric field, field-aligned current, Pedersen conductivity and total Joule heating for a run in which a 100 mV/m electric field, modeled by a Gaussian profile with a 5 km scale length, was incident on the ionosphere and held constant. This field produces a field-aligned current that is...
upward in the positive $x$ region and downward for negative values of $x$. A 10 mV/m background electric field is also present in this simulation. This run includes both the linear source term proportional to the field-aligned current and a term that is proportional to the energy flux of electrons produced by the parallel electric field in the upward current region. This field is modeled by an anomalous resistivity with $\nu^* = \nu_0 (j_\parallel / j_{crit} - 1)^2$ when $j_\parallel > j_{crit}$, where $\nu_0 = 0.1 \Omega$, and the critical current is set so that the critical drift velocity is 500 km/s. These values are suggestive of anomalous resistivity based on electrostatic ion cyclotron turbulence with a critical drift comparable to the electron thermal speed. This model and the parameters used are typical of numerical models using such resistivity [e.g., Lysak and Dum, 1983; Lysak and Hudson, 1987; Streltsov and Lotko, 1999]. The results from these models indicate that the evolution of Alfvén waves in the auroral zone is relatively insensitive to the exact parameters assumed. The energy flux for the precipitation term is the integral along the field line of $j_\parallel E_\parallel$ in regions of upward current. Note that the resistivity is applied in both upward and downward current regions; however, the energy flux contributes to the ionization only for upward currents.

[18] The results from this run show that the electron precipitation in the upward current region has given rise to an enhancement of the Pedersen conductivity up to 15 mho, while the conductivity is reduced in the downward current region due to the upward flow of electrons. The electric field and the Joule heating are reduced in the upward current region, giving a sharp edge to the electric field structure. The feedback instability in this case is driven due to the gradient in the Pedersen conductivity at the transition between the upward and downward current, and small-scale currents are produced in this region. Figure 5 shows the early time development of this run. As the run evolves and the Pedersen current is increased, the electric field is decreased in the upward current region and enhanced in the downward current region. The field-aligned current shows a flat profile due to the assumed anomalous resistivity; however, a spike of downward current forms.
near the center of the simulation at \( t = 12 \) s. It is this spike that begins the feedback interaction that acts to further reduce the electric field and Joule heating and increase the Pedersen conductivity to the point where there is a sharper conductivity gradient that gives positive feedback to the interaction.

[19] The results in Figure 4 look remarkably similar to early observations of the electrodynamics of an auroral arc from a rocket flight presented by Evans et al. [1977] that showed a transition from Joule heating to particle precipitation at the edges of an arc, while the total energy dissipation was continuous across this edge. These results show a sharp decrease of the electric field where the conductivity increased, much as in the present results. Similar observations showing a reduction in the electric field across arcs, sometimes associated with an enhancement of the electric field at the edges of the arc, have been reported from radar, rocket, and low-altitude satellite measurements [e.g., de la Beaujardière et al., 1981; de la Beaujardière and Vondrak, 1982; Marklund, 1984; Opgo-noorh et al., 1990; Johnson et al., 1998]. Such observations are clear signatures that the modification of the ionospheric conductivity is critical to auroral electrodynamics.

[20] It should be emphasized that parallel resistivity due to Coulomb collisions can be important in the ionosphere at such small scales [e.g., Forget et al., 1991; Borovsky, 1993; Lessard and Knudsen, 2001]. Although this effect is negligible in the outer magnetosphere, it can be significant in the ionosphere, particularly at lower altitudes. The resistive scale length due to parallel resistivity can be written in terms of the electron inertial length

\[
L_{\text{res}} = \sqrt{\frac{\nu_e \lambda_e}{\omega}}.
\]

where \( \nu_e \) is the electron collision frequency, including collisions with both ions and neutrals, and \( \lambda_e \) is the electron inertial length, which varies from 5 to 50 m for ionospheric densities of \( 10^6 \) to \( 10^4 \) cm \(^{-3}\). Since the electron collision frequency is about \( 10^3 - 10^4 \) s \(^{-1}\) in the ionosphere [e.g., Kelley, 1989], the resistive scale length is 30–100 times the electron inertial length for ionospheric resonator waves with \( \omega \sim 1 \) s \(^{-1}\) and 300–1000 times the electron inertial length for field line resonances with \( \omega \sim 0.01 \) s \(^{-1}\). Thus this length is 150 m \(^{-3}\) km for the ionospheric resonator and 1.5–50 km in the field line resonance case. Thus it seems likely that the
parallel resistive scale length will become important as the scale length becomes smaller. It may be noted that the simulations of Pokhotelov et al. (submitted manuscript, 2001) as well as the present simulations include the parallel resistivity effect, which not only represents a real limitation on small-scale auroral currents but also has the beneficial effect of damping potential numerical instabilities.

5. Discussion

[21] The ionospheric feedback instability provides a mechanism for the development of small-scale current structures in the auroral ionosphere. The considerations discussed above suggest that this instability results from the overreflection of Alfvén waves at the active ionosphere combined with a favorable phase shift of waves reflected back toward the ionosphere that allows the instability to grow further. It should be noted that even in the absence of the reflected waves, the overreflection implies that short wavelength fluctuations should be amplified in the presence of an active ionosphere that changes its conductivity due to particle precipitation.

[22] The presence of the feedback instability to small scales suggests that kinetic effects may become important in the evolution. Such kinetic effects would be expected to appear at the gyroradius of the electrons or ions. The electron gyroradius in a 0.5 G field is $5 \text{ cm} \times T^{1/2}$, where the temperature is given in eV and the ion gyroradius is $2 \text{ m} \times T^{1/2}$ for protons and $8 \text{ m} \times T^{1/2}$ for oxygen ions. Thus these scales are quite small for thermal ionospheric particles unless they are heated by the precipitation. Another possible mechanism for the dissipation of this instability is the excitation of shear flow instabilities [e.g., Ganguli et al., 1988; Peñano and Ganguli, 2000]. Since the feedback instability creates narrow electric field structures, the resulting $E \times B$ flows have a large shear. An important scale length for such structures occurs when the shear frequency is comparable to the ion gyrofrequency, i.e.,

$$L_{\text{shear}} = \frac{E/B}{\Omega_i} = \frac{m_i E}{eB^2}$$

For a magnetic field of 0.5 G, this scale is equal to 4 m for protons (64 m for O$^+$) and for a large assumed electric field of 1 V/m. Instability can occur in this model when the scale length is the order of 10 times this shear length. This condition may be met in some circumstances, although it appears that the resistive diffusion scale length is larger in most cases. Indeed, an observational study of this instability [Hamrin et al., 2001] was unable to find clear evidence that the instability condition was met, although observational constraints make it impossible to rule out the possibility that such small shear lengths could be present.

[23] The presence of short spatial scales in auroral currents and fields has a strong impact on low-resolution observations of parameters such as the Joule heating and the energy in particle precipitation. Ground-based instruments such as magnetometers and radar system have spatial resolutions of tens of kilometers or more, so that structures of 1 km or less produced by the feedback instability are not resolved. As noted in the 2000 GEM report on the magnetosphere-ionosphere coupling campaign (at http://www-ssc.igpp.ucla.edu/gem/gem2000.html), the Joule heating averaged over a finite spatial size is not the same as the averaged Pedersen conductivity multiplied by the square of the average electric field. Consider a situation in which the

![Figure 3. Change in total integrated Joule heating during the run of Figures 1 and 2. Note that the initial Joule heating due to the convection electric field has been subtracted.](Figure3.png)
Pedersen conductivity and the electric field consist of an average part plus an oscillating part

\[ E_{\perp} = E_{\perp 0} + \tilde{E}_{\perp} \cos k_{\perp}x \]

\[ \Sigma_{P} = \Sigma_{P 0} + \tilde{\Sigma}_{P} \cos (k_{\perp}x + \delta). \tag{32} \]

Note that the possibility of a phase shift \( \delta \) between the conductivity and the electric field is included. If the Joule heating is averaged over a spatial scale of many wavelengths, we have

\[ \langle \Sigma_{P} \tilde{E}_{\perp}^{2} \rangle = \Sigma_{P 0} E_{\perp 0}^{2} + \frac{1}{2} \Sigma_{P 0} \tilde{\Sigma}_{P}^{2} + \Sigma_{P 0} \tilde{E}_{\perp 0} \cdot \tilde{E} \cos \delta. \tag{33} \]

Only the first of the terms on the right-hand side of (33) can be measured by ground-based observations. The second term gives an enhancement due to the Joule heating of small-scale waves, while the third term is responsible for the reduction of Joule heating when the conductivity and electric field are out of phase, as can occur in the feedback instability. In addition, it should be noted that the simulations of the feedback instability as shown in Figure 1 show that the small-scale waves combine to give a large-scale reduction of the Joule heating. Thus the feedback instability has two consequences for Joule heating observations: first, that the Joule heating is reduced by the instability, and second, that some of the Joule heating that is present may be missed by observations that cannot resolve the small-scale fluctuations in the electric field.

[24] The small-scale precipitation may be more easily observable, even by low-resolution measurements. Estimates of the precipitating energy flux by consideration of the auroral luminosity [e.g., Liou et al., 2001] will not fully capture the emissions from narrow arcs, but they will tend to integrate over all the light produced, including that from these narrow arcs. Direct particle measurements from space-

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Figure 4. Results at \( t = 16 \) s of a simulation in which an electric field of 100 mV/m amplitude and 5 km width is incident on a system with an initial background electric field of 10 mV/m and an ionospheric conductivity of 1 mho. The electric field and Joule heating are reduced and the conductivity is enhanced in the upward current region, with the opposite occurring in the downward current region. The feedback interaction produces a spike in the field-aligned current at the large conductivity gradient between the two regions.
craft or rockets can have much smaller spatial resolution; for example, the results of Newell et al. [1996a, 1996b] measure widths down to about 0.055° of latitude, or roughly 6 km, while Freja [Boehm et al., 1995] and FAST [McFadden et al., 1998] are capable of making particle measurements at less than 1-km resolution. Thus it appears much easier to observe the particle precipitation on small scales than the associated Joule heating. This would imply that measurements that show increases in the particle precipitation relative to the Joule heating may reflect a situation where a great deal of small-scale structure is present in the particles and fields.

While these results suggest strongly that the feedback instability can produce narrow-scale structures, there are a number of features of the simulation that should be improved. Perhaps most importantly, the hot particle precipitation should be treated more realistically. We have estimated the particle precipitation rate by multiplying the ionospheric current times the potential drop. This procedure neglects the particle transit time from the acceleration region to the ionosphere (a 1-keV field-aligned electron travels 1 $R_E$ in about 0.3 s). This time delay may affect the evolution of the instability, especially since the phasing of the precipitation and the conductivity structure are important for the instability.

In addition, ionospheric conductivity feedback is only one of a number of processes that affect the scale size of auroral current and fields. Parallel potential drops that are a function of the field-aligned current density are diffusive and tend to smooth out small scales. For example, in the linear regime of the Knight [1973] relation the potential drop is roughly linear with the field-aligned current, while Temerin and Carlson [1998] and Rönnmark [1999] have proposed relations where the potential is proportional to the square of the current. Such diffusive current-voltage relations compete with the tendency of the feedback instability to develop small scales, as can be seen in Figures 4 and 5 where narrow currents occur only at the boundaries of the conductivity enhancement. Other factors can also affect the evolution at different spatial scales. For example, phase mixing at Alfvén speed gradients can produce small spatial scales [Lysak and Song, 2000]. In addition, nonlinear effects due to electron inertia [e.g., Seyler, 1988; Rankin and Tikhonchuk, 1998] or due to the interaction of upgoing and downgoing Alfvén wave packets [Song and Lysak, 2001] can lead to density filaments on small spatial scales.

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**Figure 5.** Early time development of the run of Figure 4 at times of 4 (solid), 8 (dotted), and 12 (dashed) seconds.
A fully nonlinear simulation including feedback effects, hot particle kinetics, inhomogeneities, and nonlinear effects would be necessary to fully describe these interactions.

[27] It should also be recognized that the electric fields and field-aligned currents are not always on the same scales. Alfvén waves at scales less than the electron inertial length become more electrostatic, as can be seen, for example, in Figure 3 of Lysak and Song [2000]. Recent observations from the Polar spacecraft [Mozer and Hull, 2001] indicate that electric fields and density structures occur on smaller scales than the field-aligned current, which would be consistent with an Alfvén wave structure with a scale smaller than the inertial length. The formation of these density structures is not included in the present MHD model of Alfvén wave propagation, although these cavities are likely formed by the acceleration of auroral electrons out of the acceleration region [e.g., Thompson and Lysak, 1996; Kletzing and Hu, 2001; Chasten et al., 2001]. Thus a complete model of the feedback instability will require a better description of the acceleration of electrons and the formation of density depletions along auroral field lines.

6. Conclusions

[28] This analysis of the feedback instability, particularly with regard to the energy source of the instability, has indicated that this instability may play an important role in the formation of small-scale auroral arcs. The theoretical considerations and numerical modeling presented in this paper lead to the following conclusions:

1. The feedback instability is effective at producing small-scale structures when the background (sunlight-produced) conductivity is low. The strong damping due to recombination as well as the tendency of conductivity enhancements to polarize rather than to produce secondary currents when the conductivity is high makes the instability weaker in daylight situations.

2. The feedback instability takes its free energy from the reduction of the background Joule heating. This can be viewed in terms of the overreflection of Alfvén wave energy in an active ionosphere, which results since the impact of an Alfvén wave pulse on the ionosphere will modify the conductivity and electric field so that the Joule heating is locally reduced, releasing energy to the reflected Alfvén wave. This overreflection can only occur when the wavelength of the wave is below a threshold set by the wave frequency and the drift velocity of the ionospheric current.

3. Narrow-scale current structures can be produced where conductivity gradients are large. In the linear feedback case (Figures 1 and 2), the conductivity gradient can steepen due to the nonlinearity in the ionospheric Ohm’s law. When ionization due to precipitation is included (Figures 4 and 5), narrow currents can form at the edge of the upward current region where there is a large conductivity gradient.

4. Large-scale measurements of the energetics of the auroral zone must be careful to take into account the contributions from small-scale structures. Estimates of the Joule heating are likely to be underestimated when small-scale electric fields are present. Particle energy fluxes may also be underestimated but not to as great a degree. Thus measurements of the ratio of the particles energy flux to Joule heating will be enhanced when small-scale structures are present.

Appendix A: Details of the Numerical Model

[29] The present model is a variation on the magnetosphere-ionosphere coupling model that has been previously presented [e.g., Lysak, 1997, 1999; Lysak and Song, 2000, 2001]. This model is based on Maxwell’s equations coupled with a linear plasma response. The perpendicular currents in the model are due to polarization and displacement currents, and are described by the dielectric constant

$$\varepsilon_\perp = \varepsilon_0 \left(1 + \frac{c^2}{p^2} \right)^{1/2}. \quad (A1)$$

Note that this form of the dielectric constant accounts for the displacement current, which results in the phase speed of the Alfvén wave being limited by the speed of light. The full Faraday and Ampere laws then give evolution equations for the magnetic and electric fields

$$\frac{\partial B}{\partial t} = - \nabla \times E \quad (A2)$$

$$\varepsilon_\perp \frac{\partial E_\perp}{\partial t} = \frac{1}{\mu_0} (\nabla \times B)_\perp \quad (A3)$$

$$\varepsilon_\parallel \frac{\partial E_\parallel}{\partial t} = \frac{1}{\mu_0} (\nabla \times B)_\parallel \cdot -j_t. \quad (A4)$$

The field-aligned current is assumed to be carried only by electrons; thus the evolution equation for this current is then essentially the electron equation of motion, which gives

$$\frac{\partial j_\parallel}{\partial t} = \frac{ne^2}{m_e} E_\parallel - \nu_\parallel j_\parallel. \quad (A5)$$

Note here that the possibility of a parallel collision frequency, whether classical or anomalous, has been included.

[30] The explicit use of the displacement current in equation (A4) is novel to this model and requires more explanation. In previous models that included electron inertial effects, the parallel displacement current, i.e., the left-hand side of equation (A4), is generally neglected. Including the displacement current explicitly not only gives the most physically meaningful equation for the parallel electric field [Parker, 1996; Song and Lysak, 2001; Vasyliunas, 2001] but also has a number of advantages numerically. In particular, additional effects such as inhomogeneities, nonlinear effects, or a kinetic plasma response could be used in place of the simplified equation (A5) to describe more complicated situations. Equations (A4) and (A5) describe plasma oscillations, which are usually at a higher frequency than is of interest in this problem. Numerically, however, the time step in the solution of these equations must be less than the inverse of the plasma frequency, which is a severe limit on the time step, particularly near the ionosphere where the density is high.
These issues can be satisfactorily resolved if the dielectric constant $\varepsilon_\parallel$ is artificially enhanced over its vacuum value $\varepsilon_0$. This would have the effect of reducing the effective speed of light, $c^* = 1/(\varepsilon_\parallel k_0)^{1/2}$ and the effective plasma frequency $\omega^* = (ne^2/m_\text{e})^{1/2}$, while leaving the important quantity, the inertial length $\lambda = c^*/\omega^*$ unchanged. It is instructive to consider the curl term in equation (A4) to be a source oscillating with a frequency $\omega$. Then, assuming that the parallel electric field and the field-aligned current also oscillate at this frequency, we can solve for the parallel electric field

$$\left(1 - \frac{\omega(\omega + i\nu_i)}{\omega^*}ight) E_\parallel = \lambda^2(\nu_i - i\omega)(\nabla \times B)_\parallel. \quad (A6)$$

It can be seen that as long as the effective plasma frequency is much greater than the wave frequency or the collision frequency, the modification of the plasma frequency does not influence the value of the parallel electric field. Thus, in practice we may pick the effective dielectric constant $\varepsilon_\parallel$ such that the effective plasma frequency is small enough so that the time step does not need to be extremely small but large enough so that the second term in the parentheses on the left-hand side of equation (A6) is negligible. Since the time step needs to be small enough to resolve the oscillations, these conditions are easy to satisfy. Typically in the runs shown, the time step is the order of $10^{-3}$ s; thus the effective plasma frequency is the order of $1$ kHz, much greater than the $1$ Hz frequencies that are of interest.

Thus the model consists of the numerical solutions of equations (A2)–(A5). These equations are cast into a quasi-dipolar coordinate system, which models an isotropic flux tube whose size expands as $r^{1.5}$, where $r$ is the radial distance in Earth radii. This model gives the correct scaling of a dipole geometry while ignoring curvature effects and the fact that a true dipole geometry, the scaling of the flux tube in latitude and longitude is different, i.e., a square in the ionosphere does not map to a square in the outer magnetosphere. This model is a good approximation to a dipole as long as the radial distance is small compared with the $L$ value of the dipole field line, which is a good approximation when processes in the auroral acceleration region are being considered. The electron density profile is of the form

$$n_e(r) = n_0 e^{-r/h} + n_1 r^{-p}, \quad (A7)$$

where the first term represents an exponential population of oxygen and the second a power law population of hydrogen. Note that $z$ is the altitude from the surface of the Earth while $r$ is the geocentric distance measured in Earth radii. Note that equation (A7) represents the electron density; for the purposes of calculating the mass density, the first term is multiplied by 16 relative to the second term. The values used in the runs in this paper are $n_0 = 5 \times 10^5$ cm$^{-3}$, $h = 160$ km, $n_1 = 10$ cm$^{-3}$, and $p = 1$.

The boundary condition at the ionosphere deserves special attention. We consider the ionosphere to be a thin slab that can carry a sheet current. If we let the positive $z$ direction be upward, and consider the electromagnetic jump conditions across such a sheet current, the current intensity $\mathbf{K}$ can be expressed as

$$\mathbf{K} = \Sigma_p \mathbf{E}_\perp + \Sigma_H \mathbf{E}_\perp \times \mathbf{z} = \frac{1}{\nu_0} \mathbf{z} \times (\mathbf{B}_\perp - \mathbf{B}_{\text{atm}}^\text{superscript}). \quad (A8)$$

where the upper (minus) sign refers to the Southern Hemisphere and the lower (plus) sign to the northern. Note that this distinction occurs because the background magnetic field is in the $\pm \mathbf{z}$ direction in the southern/northern ionosphere, respectively. Note that the magnetic field below the ionosphere in the atmosphere is denoted with the $\text{atm}$ superscript. This component is present when compressional waves are present in the system. However, it can be shown that at the small scales described here, the compressional wave is evanescent, which implies in the present two-dimensional $(xz)$ geometry that only the $E_x$, $B_y$, and $E_z$ components are non-zero, while the atmospheric field goes to zero. In this case, equation (A8) can be inverted to read

$$E_z = -\frac{\Sigma_p B_y}{\nu_0 (\Sigma_p^2 + \Sigma_H^2)} \rightarrow -\frac{B_y}{\nu_0 \Sigma_p^2}, \quad (A9)$$

where the last form holds if the Hall conductivity is neglected, as we have done in the runs shown. However, it can be seen that the results including a Hall conductivity can be included if we replace the Pedersen conductivity with the Cowling conductivity, $\Sigma_p + \Sigma_H^2/\Sigma_p$. Details of this ionospheric model in three dimensions are presented by Lysak and Song [2001].

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